FIITJEE

# ALL INDIA TEST SERIES

## FULL TEST – III

## JEE (Main)-2019

**TEST DATE: 30-12-2018**

# ANSWERS, HINTS & SOLUTIONS

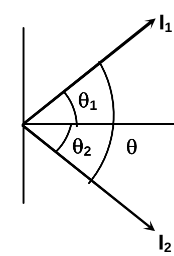
## Physics

**PART – I**

**SECTION – A**

1. C

Sol. $I_1 = \frac{50}{\sqrt{7^2 + (24)^2}} = 2\text{ A}$




$I_2 = \frac{50}{\sqrt{6^2 + 8^2}} = 5\text{ A}$

$\tan\theta_2 = \frac{6}{8} = \frac{3}{4} \Rightarrow \theta_2 = 37°$

$\tan\theta_1 = \frac{7}{24}$

$\tan\theta = \tan(\theta_1 + \theta_2) = \frac{4}{3}$

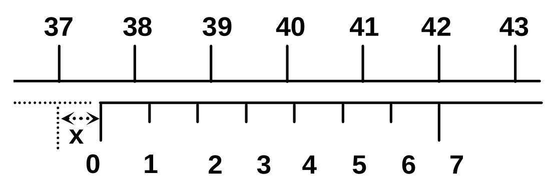
$\theta = 53°$

$I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$

$\sqrt{4 + 25 + 2\cdot 2\cdot 5\cdot \cos 53°} = \sqrt{41}$

2. A

Sol. 10 V = 7S




V = 0.7 S

So, 7V = 4.9 S

X + 7V = 5S

X = 5S – 4.9 S

= 0.01 cm

So, measured value = 3.7 + 0.01 = 3.71 cm

3. A

Sol. $\beta = \frac{\Delta I_c}{\Delta I_B} = \frac{2mA}{20\mu A} = 100$

Voltage gain = $\frac{2mA \times 5k\Omega}{20\ mV} = 500$

Resistance gain = $\frac{\text{Voltage gain}}{\text{Current gain}} = \frac{500}{100} = 5$

So, Input resistance = $\frac{\text{Output resistance}}{5} = \frac{5\ k\Omega}{5} = 1\,k\Omega$

4. C

Sol. $y_1 = \frac{n_1\lambda_1 D}{d} = y_2 = \frac{n_2 y_2 D}{d}$

$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{980}{560} = \frac{7}{4}$

$y_1 = \frac{7 \times 560 \times 10^{-9} \times 1.5}{1 \times 10^{-3}} = 5.88\ \text{mm}$

$n = \frac{17.64}{5.88} = 3$

5. D

Sol. $\phi = EA$

$\phi = 40\ x^3 A$

$d\phi = (120\ x^2)Adx$

$dq = (d\phi)\varepsilon_0 = (120\ \varepsilon_0 x^2)(Adx)$

$\rho(x) = \frac{dq}{dv} = \frac{120\varepsilon_0 x^2}{(Adx)}(Adx)$

$\rho(x) = 120\ \varepsilon_0 x^2$

$\rho(x = 2) = 480\ \varepsilon_0$

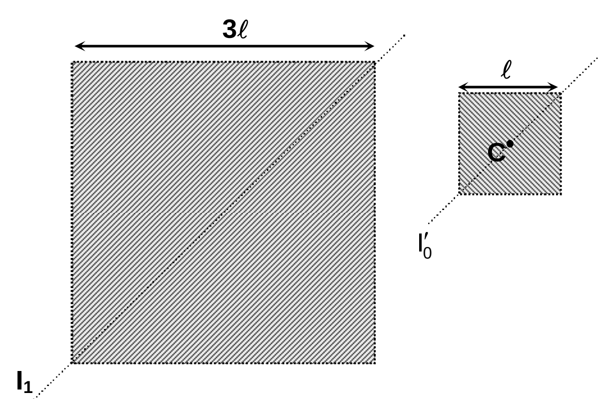
6. B

Sol. $I_C = I_0$

$I_0' = \frac{I_0}{2} = K\ell^4$

$I_1 = k_0(3\ell)^4 = 81\left(k_0\ell^4\right) = \frac{81}{2}I_0$

$I_{AB} = \frac{\left(\frac{81}{2}I_0 - \frac{3}{2}I_0\right)}{2} = \frac{39}{2}I_0$




7. D

Sol. Fundamental frequency of open pipe is

$f_1 = \frac{v}{2\ell} = \frac{292}{2 \times (0.5)} = 292\ \text{Hz}$

$\ell_f = \ell_i + \Delta\ell = (0.98 + 0.02) = 1.0\ \text{m}$

Fundamental frequency of stressed wire

$f_2 = \frac{1}{2\ell_f}\sqrt{\frac{T}{\pi r^2 \rho}}$ …(i)

$f_2 = f_1 \pm 8 \Rightarrow f_2 = 300$ Hz or 284 Hz

As the number of beats decreases on decreasing the stress so $f_2 = 300$ Hz

So from (i) $\frac{T}{\pi r^2} = 4f_2^2 \ell_f^2 \rho$

$y = \frac{(T/\pi r^2)}{(\Delta\ell/\ell_i)} = \frac{4f_2^2\ell_f^2\rho}{(\Delta\ell/\ell_i)} = \frac{4(300)^2(1)^2 \times 10^4}{(0.02)/(0.98)}$

$= 17.64 \times 10^{10}$ N/m$^2$

8. C

Sol. $i = \sqrt{4}$ amp

$\frac{q_m^2}{2C} - \frac{q^2}{2C} + \frac{1}{2}LC^2$

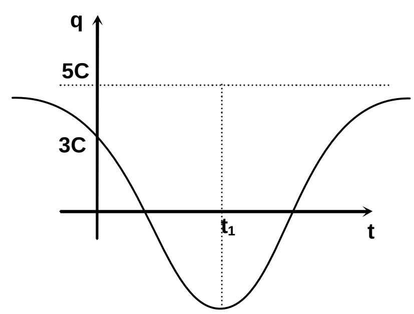
$q_m^2 = 9 + 4(\sqrt{4})^2$

$q_m = 5C.$

$q = Q\cos(\omega t + \delta)$

at $t = 0$

$3 = 5\cos\delta \Rightarrow \delta = 53°$




Let at $t = t_1$, charge on capacitor is maximum

$\omega t_1 + \delta = \pi$

$t_1 = \left(\frac{\pi - \delta}{\omega}\right) = (\sqrt{LC})(\pi - \delta) = \frac{2(180 - 53°)\pi}{180} = \left(\frac{127\pi}{90}\right)$sec

9. C

Sol. Bulk modulus $= \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$K = \frac{F/A}{dV/V} \Rightarrow \frac{F/A}{K} = \frac{dV}{V}$

$V = \ell^3$, $dV = 3\ell^2 D\ell$

$\frac{dV}{V} = \frac{3d\ell}{\ell}$

$\frac{3d\ell}{\ell} = \frac{F}{KA} \Rightarrow \frac{d\ell}{\ell} = \left(\frac{F}{3KA}\right)$

10. B

Sol. $q = \frac{\Delta\phi}{R}$

$\Delta\phi = Rq = R\int i\,dt$

$= (200)\left(20 \times 1 + \frac{20 \times 1}{2}\right) = 6000\text{Wb}$

$\varepsilon_{av} = \frac{\Delta\phi}{\Delta t} = \frac{6000}{2} = 3000\text{ V}$

11. B

Sol. Loss of weight at 28°C = (40 – 32) × $10^{-3}$g = $v_1 \cdot 2\rho_\ell g$ …(i)

Loss of weight at 32°C = (40 – 28) × $10^{-3}$g = $v_2 \cdot (1.5\rho_\ell)g$ …(ii)

From (i) and (ii)

$\frac{V_1}{V_2} = \frac{1}{2}$ …(iii)

$V_2 = V_1(1 + 3\alpha\Delta T)$

$\alpha$ = (1/12)/°C

12. A

Sol. Optical source frequency $f = \frac{c}{\lambda} = \frac{3\times10^8}{1.5\times10^{-6}} = 2\times10^{14}$

Number of channels or subscribers $= \frac{2\times10^{14}}{25\times10^3} = \frac{200\times10^{12}}{25\times10^3} = 8\times10^9$

13. C

Sol. Charge in momentum

$\Delta P = \int F dt$

$\Delta P = \int_0^3 (3t^2 + 6t)dt = 54$

$\Delta K = \frac{P^2}{2m} = \frac{54\times54}{2\times18} = 81 \text{Joule}$

14. D

Sol. Let $A_1 = A_0$, $A_2 = 3A_0$

Then $A_{max}^2 = A_0^2 + 9A_0^2 + 2A_0\cdot3A_0 = 16A_0^2$

$A_{max} = 4A_0$ …(i)

For $\phi$ = 120

$A^2 = A_0^2 + 9A_0^2 + 2A_0\cdot3A_0\cos120° = 10A_0^2 - 3A_0^2 = 7A_0^2$

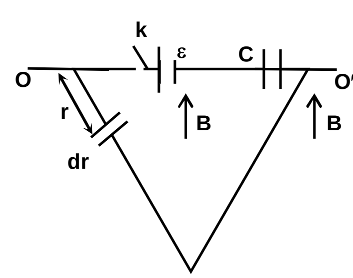
$A = \sqrt{7}A_0$ …(ii)

$\frac{I}{I_{max}} = \frac{I}{I_0} = \frac{7A_0^2}{16A_0^2}$

$I = \frac{7}{16}I_0$

15. C

Sol. Torque about OO′ $\tau = (idr\cos60°)Br\sin60°$




$= \frac{\sqrt{3}}{4}iB\int_0^\ell rdr = \frac{\sqrt{3}}{4}iB\ell^2$

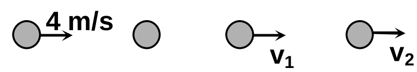
Angular impulse, $\int \tau dt = I\omega$

$\int \frac{\sqrt{3}}{4}iB\ell^2 dt = \frac{m\ell^2}{2}\omega$

$\frac{\sqrt{3}}{4}B\ell^2\int idt = \frac{m\ell^2}{2}\omega$

$$\frac{\sqrt{3}}{4}B\ell^2C\varepsilon = \frac{m\ell^2}{2}\omega$$
$$\omega = \frac{\sqrt{3}BC\varepsilon}{2m}$$

16. D




Sol. $\frac{1}{2}4v_1^2 + \frac{1}{2}4v_2^2 = \frac{1}{2}\left(\frac{1}{2}4\cdot 4^2\right)$

$v_1^2 + v_2^2 = 8$ …(i)

$4v_1 + 4v_2 = 4 \times 4$

$v_1 + v_2 = 4$ …(ii)

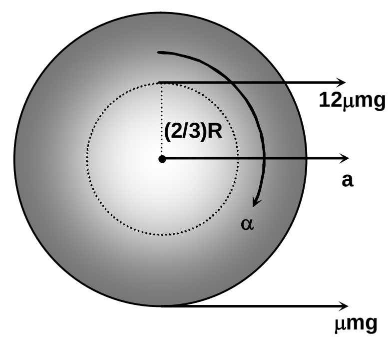
from (i) and (ii)

$v_1 = v_2 = 2$ m/s

Hence $e = \frac{v_2 - v_1}{4} = 0$

17. D

Sol. $r = \frac{I_{cm}}{mR} = \frac{\frac{mR^2}{2}}{mR} = \frac{R}{2}$



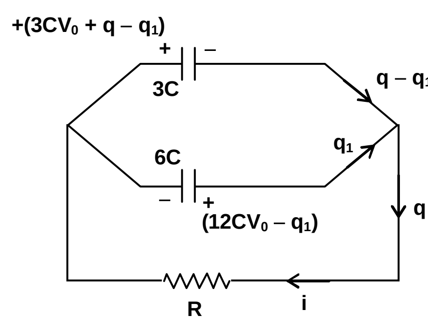

But force acts above to it so friction on the disc acts along the applied force.

Torque about centre of mass

$$\frac{mR^2\alpha}{2} = 12\mu mg\left(\frac{2}{3}R\right) - \mu mgR = 7\mu mgR$$
$$\alpha = \frac{14\mu g}{R}$$

18. D

Sol. Applying Kirchhoff' law




$$-\frac{3CV_0 + q - q_1}{3C} - iR = 0 \quad \ldots(i)$$
$$\frac{12CV_0 - q_1}{6C} - iR = 0 \quad \ldots(ii)$$

From (i) and (ii)

$q = 9CV_0\,(1 - e^{-t/(9RC)})$

19. C

Sol. $K_1 = \frac{hC}{\lambda} - \phi = eV$ …(i)

$K_2 = \frac{2hC}{\lambda} - \phi = e3V$ …(ii)

From (i) and (ii)

$$\left(\frac{hC}{\lambda} - \phi\right)3 = \left(\frac{2hC}{\lambda} - \phi\right)$$
$$\frac{3hC}{\lambda} - 3\phi = \frac{2hC}{\lambda} - \phi$$

$$\frac{hC}{\lambda} = 2\phi \Rightarrow \phi = \frac{hC}{2\lambda} \quad \ldots\text{(iii)}$$

$$\frac{hC}{\lambda/3} - \phi = eV_0$$

$$\frac{hC}{\lambda/3} - \frac{hC}{2\lambda} = eV_0$$

$$\frac{hC}{\lambda}\left(3 - \frac{1}{2}\right) = eV_0 \Rightarrow V_0 = \left(\frac{5hC}{2e\lambda}\right)$$

20. A

Sol. $T = 2\pi\sqrt{\frac{I}{mgd}}$

$$= 2\pi\sqrt{\frac{\left(\frac{5}{3}mR^2 + \frac{mR^2}{4} + mR^2\right)}{(2m)(g)\left(\sqrt{R^2 + \frac{R^2}{16}}\right)}}$$

$$= \left(2\pi\sqrt{\frac{35R}{6\sqrt{17}g}}\right)$$

21. B

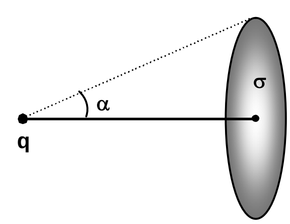
Sol. Total upward force due to surface tension = weight of the liquid column of height h.

$$T \cdot 2\pi R + T \cdot 4r = \rho\left(\pi R^2 - r^2\right)gh$$

$$h = \frac{2T(\pi R + 2r)}{(\pi R^2 - r^2)\rho g}$$

22. B

Sol. Electric field due to disc is




$$E = \frac{\sigma}{2\epsilon_0}(1 - \cos\alpha) \quad \ldots\text{(i)}$$

Flux through the disc is

$$\phi = \frac{q}{2\epsilon_0}(1 - \cos\alpha) \quad \ldots\text{(ii)}$$

Force due to disc on q

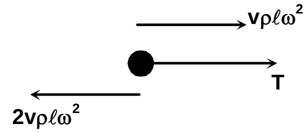
F = qE

$$= \frac{\phi.2\epsilon_0}{(1 - \cos\alpha)}.\frac{\sigma}{2\epsilon_0}(1 - \cos\alpha)$$

$$= \sigma\phi$$

23. B

Sol. $P_1 = P_D + \frac{\rho\omega^2\ell^2}{2}$




$$P_2 = P_D + \rho\frac{\omega^2\ell^2}{2} + \rho\ell g + \frac{3}{2}\rho\omega^2\ell^2$$

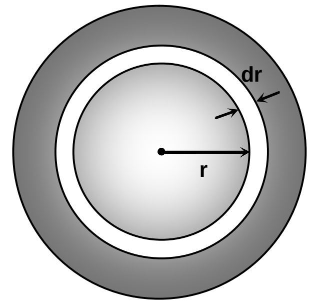
$$P_2 - P_1 = 2\rho\ell g$$

$\therefore \qquad \omega = \sqrt{\frac{2g}{3\ell}}$

So, T = $2v\rho\ell\omega^2 - v\rho\ell\omega^2$ = (4/3)mg

24. A

Sol. di = $\frac{dq}{2\pi/\omega} = \frac{\omega}{2\pi}(\sigma, 2\pi r.dr)$




$$B = \int_0^6 \frac{\mu_0}{2} \frac{\left(\frac{\omega}{2\pi}.2\pi r\left(Ar - Br^2\right)dr\right)}{r} = 0$$

$$\int_0^6 \left(Ar - Br^2\right)dr = 0$$

$$\left(\frac{Ar^2}{2} - \frac{Br^3}{3}\right) = 0$$

$$\frac{A}{B} = \frac{2}{3}r = \frac{2}{3}.6 = 4$$

25. B

Sol. $\rho = \frac{M}{V} = \frac{M}{\pi r^2 \ell}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 2\left(\frac{\Delta r}{r}\right) + \frac{\Delta\ell}{\ell}$$

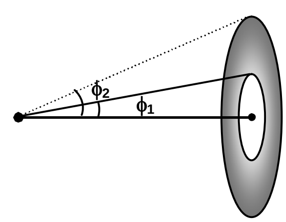
$$= \frac{0.003}{0.3} + 2 \times \left(\frac{0.005}{0.5}\right) + \left(\frac{0.006}{0.6}\right) = 0.04$$

Percentage error = $\frac{\Delta\rho}{\rho} \times 100 = 4\%$

26. C

Sol. $F = \left[2\pi G\sigma\left(\cos\phi_1 - \cos\phi_2\right)m\right]$

$$= \frac{Gmm}{65R^2}$$




27. A

Sol. $\frac{\Delta\lambda}{\lambda} = \frac{\lambda + \Delta\lambda}{65}$

$$\frac{\Delta\lambda}{\lambda(\lambda+\Delta\lambda)} = \frac{(\lambda+\Delta\lambda)-\lambda}{\lambda(\lambda+\Delta\lambda)} = \frac{1}{\lambda} - \frac{1}{\lambda+\Delta\lambda} = \frac{1}{65}$$

$$f_1 - f_2 = 5$$

$$\frac{v}{\lambda} - \frac{v}{\lambda+\Delta\lambda} = 5$$

From (i) and (ii)

$$\frac{5}{v} = \frac{1}{65}$$

v = 325 m/s

28. B

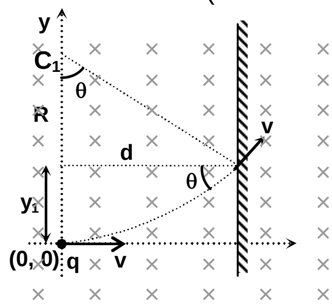
Sol. $y_1 = (R - R\cos\theta)$

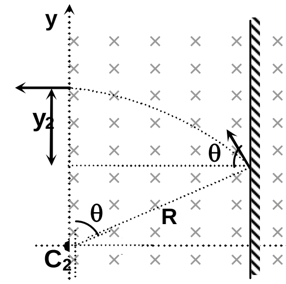
$y_2 = R - R\cos\theta$

So, y-coordinate

$y_0 = y_1 + y_2$

$= 2R - 2R\cos\theta$ ($\sin\theta = d/R = 4/5 \Rightarrow \theta = 53°$)




$= 2R - 2R\left(\frac{3}{5}\right)$ s

$= \frac{4R}{5} = \frac{4mv}{5qB_0}$

29. C

Sol. Assume both moving pullies are in rest

$a_{20kg} = \frac{200-100}{30} = \frac{10}{3}$ m/s$^2$

So, Tension $T_1 = 200 - 20 \times \frac{10}{3} = \frac{400}{3}$N

$a_{15\,kg} = \frac{150-120}{27} = \frac{30}{27} = \frac{10}{9}$ m/s$^2$

So, tension $T_2 = 150 - \frac{150}{9} = \frac{400}{3}$N

$T_1 = T_2$

So, both pulley remains in rest.

So, $T_s = 2T_1 + 2T_2$

$= 4\left(\frac{400}{3}\right) = \frac{1600}{3}$N

30. C

Sol. $x = 2t^2$, $y = 3t^2$

$v_x = 4t$, $v_y = 9t^2$

$\tan\theta = \frac{v_y}{v_x} = \frac{9}{4}t$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{9}{4}$

$\frac{d\theta}{dt} = \frac{9/4}{1+\tan^2\theta} = \frac{9/4}{1+\frac{81}{16}t^2}$

at t = 4, $\frac{d\theta}{dt} = \frac{9}{4} \times \frac{1}{82} = \frac{9}{328}$

# Chemistry

## PART – II

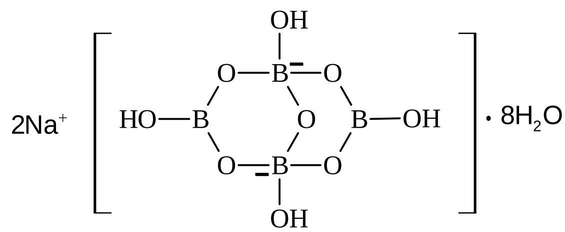
### SECTION – A

31. A

Sol. $2MnO_4^- + H_2O + I^- \longrightarrow 2MnO_2 + 2OH^- + IO_3^-$

32. D

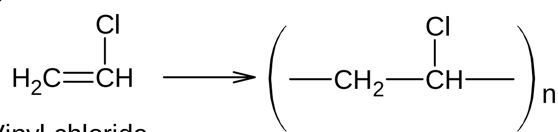
Sol.



$B_2H_6 + 6H_2O \longrightarrow 2H_3BO_3 + 6H_2 \uparrow$

33. C

Sol.



Vinyl chloride PVC

PVC is a plastic.

34. B

Sol. $IF_7$ has $sp^3d^3$ hybridisation.

35. C

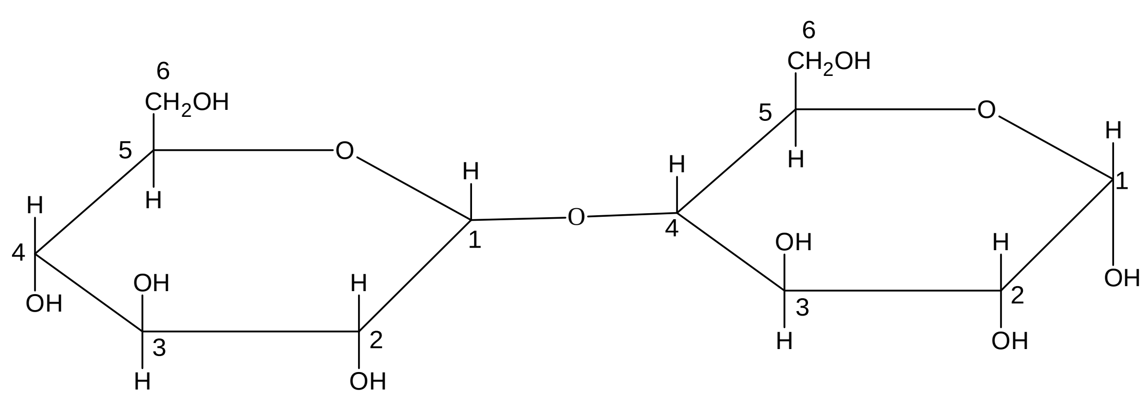
Sol. $nH_2 = 8/2 = 4$

$nSO_2 = \frac{32}{64} = \frac{1}{2}$

$\frac{P_{H_2}}{P_{SO_2}} = \frac{nH_2}{nSO_2} = 8$

36. A

Sol.



37. B

Sol. In the given process temperature, first increases then decreases.

38. B
Sol. P can form $d\pi$ - $d\pi$ bond with transition metals.

39. D
Sol. $\left[H^+\right] = 10^{-3}$

$$K_{a_2} = \frac{\left[H^+\right]\left[HA^{2-}\right]}{\left[H_2A^-\right]} \qquad K_{a_3} = \frac{\left[H^+\right]\left[A^{3-}\right]}{\left[HA^{2-}\right]}$$

40. A

41. D
Sol. Accepted transitions
1. $5 \rightarrow 1$
2. $5 \rightarrow 4 \rightarrow 1$
3. $5 \rightarrow 3 \rightarrow 1$
4. $5 \rightarrow 4 \rightarrow 3 \rightarrow 1$

42. C

43. C
Sol. $\left[Ni(CN)_4\right]^{2-} \longrightarrow dsp^2$

$\left[NiCl_4\right]^{2-} \longrightarrow sp^3$

$\left[Fe(Co)_5\right] \longrightarrow dsp^3$

$\left[Ag(NH_3)_2\right]^+ \longrightarrow sp$

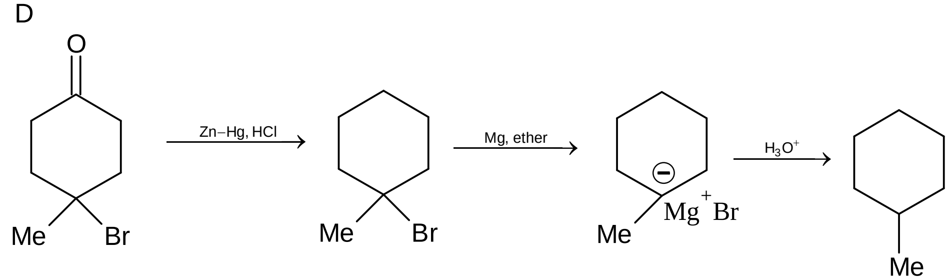
$\left[Co(H_2O)_6\right]^{3+} \longrightarrow d^2sp^3$

$CrO_4^{2-} \longrightarrow d^3s$

44. D

45. C

46. B

47. D
Sol.



48. D

49. D
Sol. $mc\ \Delta T = 20000 \times \text{Mass of HC}$

$$\text{Mass of HC} = \frac{5 \times 50 \times 4200}{20000}$$

= 52.5 g

50. B
Sol. All salts are of form PQ

$$PQ(s) \rightleftharpoons P^{+}(aq) + Q^{-}(aq)$$
$$\qquad\qquad S \qquad S$$

$K_{sp} = S.\ S = S^2$

$$e^{-} + PQ(s) \rightleftharpoons P(s) + Q^{-}(aq.)$$

$$E = E^{o} - \frac{0.059}{1}\log\left[Q^{-}\right]$$

E = 0

$$E^{o} = \frac{0.059}{2}\log K_{sp}$$

51. C
Sol. $R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = \frac{1}{\lambda}$

$$n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

52. C
Sol. $\frac{x}{m} \propto P^{1/n}$

$$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$$

$$\frac{1}{n} = \tan 30^{o} \qquad P = 1\,atm$$

$$\log\frac{x}{m} = \log K$$

As log K = 0.477

$\frac{x}{m} = 3g$ per 1 g of adsorbent

53. D
Sol. $\Delta H_{reaction} = Ea_{(f)} - Ea_{(b)}$

$$q - p = Ea_{(f)} - r$$

54. D
Sol. $SiO_2$ in bauxite
Si in pig iron

55. A
Sol. $SN^1$ reactivity depends upon stability of carbocation.

56. C
Sol. $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

57. A
Sol. $a_\alpha$ = edge length of $\alpha$ form
$a_\beta$ = edge length of $\beta$ form

$$\frac{a_\alpha}{a_\beta} = \frac{\sqrt{3}}{\sqrt{2}}$$

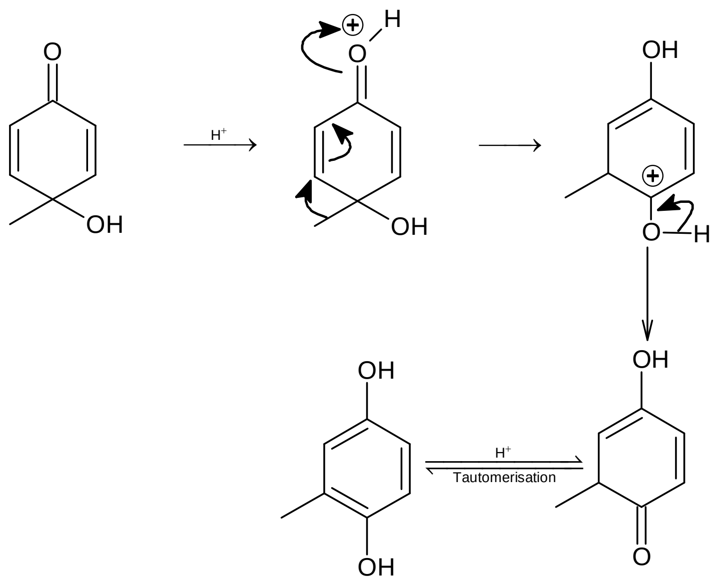
58. D

59. C

Sol. $SiO_2 + 4HF \longrightarrow SiF_4 + 2H_2O$

60. B

Sol.



# Mathematics

## PART – III

## SECTION – A

61. B

Sol. $M = \frac{10!}{4!(2!)^2}$; $N = \frac{6!}{(2!)^2} \times {}^7C_4$; $\frac{M}{N} = \frac{10!}{4!(2!)^2} \times \frac{(2!)^2\, 4! \times 3!}{6! \times 7!} = \frac{10 \times 9 \times 8}{5!} = 6$

62. B

Sol. Put $\tan^{-1}\left(x^2 + \frac{1}{x^2}\right) = t$

63. B

Sol. Circumcentre is point of intersection of perpendicular bisector of sides and perpendicular bisector an

$x + y + 7 = 0$

$x - 2y + 4 = 0$

Circumcentre is (–6, –1)

64. B

Sol. Let $x = \frac{1-u}{1+u}$

$I = \int_0^1 \frac{\ln 2 - \ln(1+u)}{1+u^2}$

$2I = \ln 2\left[\tan^{-1} u\right]_0^1$

$I = \frac{\pi \ln 2}{8}$

65. A

Sol. Vector perpendicular given lines $\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & 1 & -3 \end{vmatrix}$

$\hat{i}(3-1) - \hat{j}(-3) + \hat{k}(1-0) = 2\hat{i} + 3\hat{j} + \hat{k}$

Required line is $\vec{r} = \left(\hat{i} + 2\hat{j} - k\right) + \lambda\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

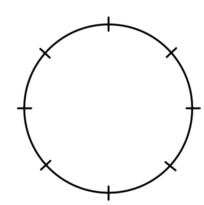
66. A

Sol. Let $m_1$ and $w_1$ always sit together

Number of ways to sit 8 men on normal table = 7!

$w_1$ can be sited in 2 ways and rows 7 women can sited in 7! ways

Hence, Rearranged number = $7! \times 2 \times 7! = 2 \times (7!)^2$




67. B

Sol. As $f(k - x) = f(k + x)$

Hence, f(x) is symmetric about x = k

If $\alpha_1 = p - \lambda_1$ then $\alpha_2 = p + \lambda_2$

$\alpha_3 = p - \alpha_1$ then $\alpha_4 = p + \lambda_4$
⋮
⋮
$\alpha_1 + \alpha_2 + \alpha_3 + ..... + \alpha_n = nk \; \forall \; n \in N$

68. A
Sol. For skew-symmetric matrix
$a_{ij} = -a_{ji}$
Only six positions are to be filled using $\{\pm4, \pm3, \pm2, \pm1, 0\}$
Number of ways = $9^6$

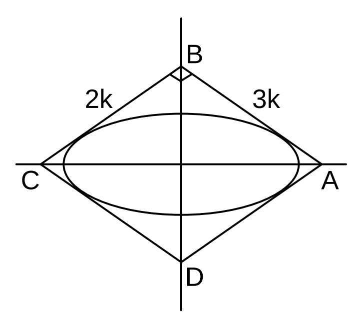
69. C
Sol. $-1 < g(x) < 1$
f(g(x)) is Bijective in $\forall\, x \in R$ and also f(g(x)) is differentiable

70. B
Sol. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = \dfrac{\cos^2 2x + 3}{1 - \dfrac{(\sin 2x)^2}{4}} = 4$
Now, $x^2 + 3 = 4$; $x = \pm1$

71. C
Sol. $(1+\omega)^{100} = {}^{100}C_0 + {}^{100}C_1\omega + {}^{100}C_2\omega^2 + {}^{100}C_3\omega^3.....$
$(-\omega^2)^{100} = \left({}^{100}C_1 + {}^{100}C_3 + .....\right) - \frac{1}{2}\left({}^{100}C_1 + {}^{100}C_2 + .....\right) + \frac{i\sqrt{3}}{2}\left({}^{100}C_1 - {}^{100}C_2 - .....\right)$
$\left|(-\omega^2)^{100}\right| = 1 = \left(a - \frac{b}{2}\right)^2 + \frac{3}{4}c^2$

72. A
Sol. $\min\left[(x_1 - x_2)^2 + \left(7 + \sqrt{4 - (x_1+4)^2} - \sqrt{4x_2}\right)^2\right]$ = shortest distance between the curve
$(x + 4)^2 + (y - 7)^2 = 4$ and $y^2 = 4x$

73. B
Sol. Vertices of rectangle lies on director circle of
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$




Equation of director circle $x^2 + y^2 = a^2 + b^2$
$AC = 2\sqrt{a^2 + b^2}$
$AC^2 = AD^2 + CD^2 = AB^2 + BC^2$
$2\sqrt{a^2 + b^2} = \sqrt{(2k)^2 + (3k)^2}$
$\Rightarrow 4(a^2 + b^2) = 13k^2$; $k^2 = \frac{4(a^2 + b^2)}{13}$; Area of rectangle = $3k \times 2k = 6k^2 = \frac{24(a^2 + b^2)}{13}$

74. C

Sol. $-(p \leftrightarrow q) \wedge p$

| **p** | **q** | **$p \leftrightarrow q$** | **$-(p\leftrightarrow q)$** | **q** | **$p \wedge -q$** |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | F | T | T | T |
| F | T | F | T | F | F |
| F | F | T | F | F | F |

75. C

Sol. $\Sigma x_i^2 = \left(\Sigma x_i\right)^2 - 2\Sigma x_i x_j = 300$ ; $\frac{\Sigma x_i^2}{10} = 30$

$\sigma = \sqrt{\frac{\Sigma x_i^2}{10} - \left(\frac{\Sigma x_i}{10}\right)^2}$ ; $\sigma = \sqrt{30-25} = \sqrt{5}$

76. B

Sol. Center of hyperbola is (–1, –1) equation of pair of asymptotes is
$5x^2 + 23xy - 10y^2 + 33x + 3y + \lambda = 0$ where passing through (–1, –1)
$\therefore \lambda = 18$

77. B

Sol. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a}\times\left(\left(\vec{x}-\vec{b}\right)\times\vec{a}\right) = (\vec{a}\cdot\vec{a})\vec{x} - (\vec{a}\cdot\vec{a})\vec{b} - (\vec{a}\cdot\vec{x})\vec{a}$

Similarly $\vec{b}\times\left((\vec{x}-\vec{c})\times\vec{b}\right) = \left(\vec{b}\cdot\vec{b}\right)\vec{x} - \left(\vec{b}\cdot\vec{b}\right)\vec{c} - \left(\vec{b}\cdot\vec{x}\right)\vec{b}$

$\vec{c}\times\left((\vec{x}-\vec{a})\times\vec{c}\right) = (\vec{c}\cdot\vec{c})\vec{x} - (\vec{c}\cdot\vec{c})\vec{a} - (\vec{c}\cdot\vec{x})\vec{c}$

Now, $\vec{a}\times\left(\left(\vec{x}-\vec{b}\right)\times\vec{a}\right) + \vec{b}\left(\left(\vec{x}-\vec{b}\right)\times\vec{b}\right) + \vec{c}\times\left((\vec{x}-\vec{c})\vec{c}\right) = 0$

$\Rightarrow 3\vec{x} - \left(\vec{a}+\vec{b}+\vec{c}\right) - \left((\vec{a}\cdot\vec{x})\vec{a} + \left(\vec{b}\cdot\vec{x}\right)\vec{b} + (\vec{c}\cdot\vec{x})\vec{c}\right) = 0$ as $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vector

$\therefore \vec{x} = x_1\vec{a} + x_2\vec{b} + x_3\vec{c}$

$\vec{a}\cdot\vec{x} = x_1$; $\vec{b}\cdot\vec{x} = x_2$ ; $\vec{c}\cdot\vec{x} = x_3$

$\vec{x} = (\vec{a}\cdot\vec{x})\vec{a} + \left(\vec{b}\cdot\vec{x}\right)\vec{b} + (\vec{c}\cdot\vec{x})\vec{c}$

$3\vec{x} - \left(\vec{a}+\vec{b}+\vec{c}\right) - \vec{x} = 0$

$\vec{x} = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$

78. D

Sol. Normal vector is $\begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = i(2+3) - j(-1-6) + k(-1+4)$ = 5i + 7j + 3k

Required plane is $(\vec{r}-\vec{a})\cdot\vec{n} = 0$

$\left(\vec{r} - (i+2j-2k)\right)\cdot(5i+7i+3k) = 0$

$\Rightarrow 5x + 7y + 3z = 5 + 14 - 6; 5x + 7y + 3z = 13$

Distance of plane form (1, 1, 3) is $\left|\frac{5+7+9-13}{\sqrt{5^2+7^2+3^2}}\right| = \frac{8}{\sqrt{83}}$

79. C

Sol. $\frac{2!\left({}^{8}C_{6}\right)}{8^{6}}$

80. B

Sol. Let $S = x^2 + y^2 + 2gx + 2fy + c = 0$

$S_1 : x^2 + y^2 - 9 = 0$

$S_2 : x^2 + y^2 - 2x + 4y + 2 = 0$

Common chord of S and $S_1$ passing through (0, 0)

Common chord of S and $S_2$ passing through (1, 2)

$S - S_1 = 0$

$2gx + 2fy + c = 9 = 0 \Rightarrow c = -9$

$S - S_2 = 0$

$2gx + 2x + 2fy + 4y + c - 2 = 0$

This line passing through (1, 2)

$(2g + 2) \times 1 + (2f + 4) \times 2 + (-9) - 2 = 0$

$2g + 2f + 10 - 11 = 0$

$2g + 2f - 1 = 0$

Locus of centre $2x + 2y - 1 = 0$

81. C

Sol. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+2}-ax-b\right)=2$

$a = 1$; $b = \frac{-3}{2}$ required equation is $(x - 1)^2 + (y + 3)^2 = 1$

82. D

Sol. Any point on curve $x^2 + 4y^2 = 4$ is $P(2\cos\theta, \sin\theta)$

$PA = \left|\frac{2\cos\theta+2\sin\theta+12}{\sqrt{5}}\right|$; PA is minimum if $\theta=\frac{5\pi}{4}$; $P\left(-\sqrt{2},-\frac{1}{\sqrt{2}}\right)$

83. B

Sol. $\frac{dy}{dx}=6x^2+6=12$; $x = \pm 1$

Points on curve are (1, 13), (–1, –3)

Equation of line are $x + 12y - 157 = 0$; $x + 12y + 37 = 0$

84. C

Sol. $\frac{\left(1^3+1^3+2^3+(3!)^3.....\right)+\left(1^2+1^2+2^2+(3!)^2.....\right)+(1+1+2+6+24+120).....}{36}$

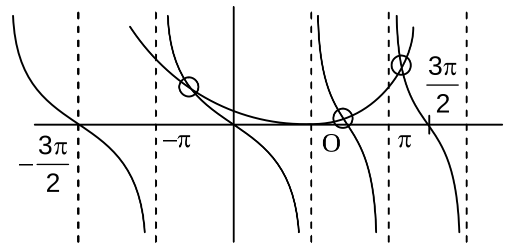
$\frac{k\times 36+26}{36}$

85. B

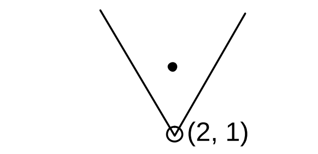
Sol. $I_1=\int_{-5}^{6}\frac{dx}{\left(6+2x-2x^2\right)\left(1+e^{3-6x}\right)}$; $I_1=\int_{-5}^{6}\frac{\left(e^{3-6x}\right)dx}{\left(6+2x-2x^2\right)\left(1+e^{3-6x}\right)}$ f(x) = f(a + b – x)

$2I_1 = I_2$; $\frac{I_1}{I_2}=\frac{1}{2}$

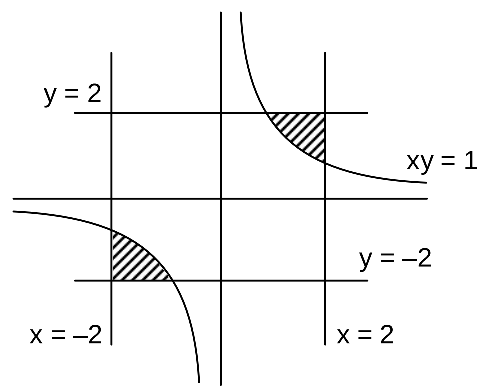
86. C
Sol. Shown in the figure




87. B
Sol. $f(2^-) < f(2)$
$f(2^+) < f(2)$




88. C
Sol. $\max(|x|, |y|) \le 2$
$\Rightarrow |x| \le 2$ and $|y| \le 2$
Required area is




$$2\int_{1/2}^{2}\left(2-\frac{1}{x}\right)dx = \left[2\left(2x - \ln x\right)\right]_{1/2}^{2}$$
$$(4 \times 2 - 2\ln 2) - \left(4\times\frac{1}{2} - 2\ln\left(\frac{1}{2}\right)\right)$$
$8 - 2\ln 2 - 2 + 2(\ln 1 - \ln 2)$
$6 - 4\ln 2$

89. C
Sol. $y = f(x)$
$f(x)\sin 2x + \sin x - (1 + \cos^2 x)f'(x) = 0$
$$\frac{dy}{dx} - y\frac{\sin 2x}{1+\cos^2 x} = \frac{\sin x}{1+\cos^2 x}$$
$$\text{I.f} = e^{-\int \frac{\sin 2x}{1+\cos^2 x}dx} = 1+\cos^2 x$$
$$y\left(1+\cos^2 x\right) = \int \frac{\sin x\left(1+\cos^2 x\right)}{1+\cos^2 x}dx = -\cos x + c$$
$$y = \frac{-\cos x + c}{1+\cos^2 x}$$
As $y(0) = 0$; $c = 1$
$$y = \frac{1-\cos x}{1+\cos^2 x}$$

90. D
Sol. f(x) is polynomial of define n, then f(f(x)) is polynomial of degree 2n and
$x\int_0^x f(t)dt$ is polynomial of degree n + 2
As $f(f(x)) = x\int_0^x f(t)dt$ ..... (1)

2n = n + 2
n = 2
f(x) is polynomial of degree 2
$f(x) = ax^2 + bx$ as f(0) = 0
Differential (1) w.r.t. x

$$f'(f(x)) \cdot f'(x) = \int_0^x f(t)dt + x f(x)$$

f′(0) = 0
Hence, $f(x) = ax^2$

Now, $f(f(x)) = x\int_0^x f(t)dt$

$$a(ax^2)^2 = x\int_0^x at^2 dt = \frac{ax^4}{3}$$

$$a^3 = \frac{a}{3} \Rightarrow a = 0, \pm \frac{1}{\sqrt{3}}$$

$$f(x) = \frac{x^2}{\sqrt{3}}$$