FIITJEE

FULL TEST – 1
JEE MAIN

DATE: 02/01/2019

ANSWERS KEY

PHYSICS

1.	4	9.	4	17.	1	25.	2
2.	2	10.	2	18.	4	26.	4
3.	3	11.	4	19.	2	27.	2
4.	1	12.	4	20.	1	28.	2
5.	3	13.	4	21.	4	29.	2
6.	4	14.	2	22.	4	30.	4
7.	1	15.	3	23.	3		
8.	4	16.	3	24.	1		

CHEMISTRY

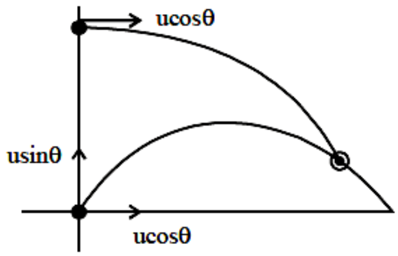
31.	1	39.	4	47.	1	55.	2
32.	4	40.	2	48.	4	56.	3
33.	4	41.	2	49.	2	57.	1
34.	4	42.	3	50.	2	58.	4
35.	2	43.	4	51.	2	59.	4
36.	2	44.	1	52.	1	60.	4
37.	1	45.	4	53.	2		
38.	4	46.	3	54.	4		

MATHEMATICS

61.	2	69.	1	77.	4	85.	4
62.	4	70.	4	78.	2	86.	3
63.	3	71.	3	79.	2	87.	3
64.	2	72.	3	80.	2	88.	1
65.	2	73.	2	81.	2	89.	2
66.	4	74.	3	82.	4	90.	2
67.	3	75.	3	83.	3		
68.	2	76.	4	84.	2		

HINTS & SOLUTIONS
PHYSICS

1. 4



$$\frac{h}{u\sin\theta} \le \frac{2u\sin\theta}{g} \Rightarrow h \le \frac{2u^2\sin^2\theta}{g} = 4H$$

2. 2

Surface Area = 2 [ab + bc + ca]
= 2 [1.5 × 1.5 + 1.5 × 1.0 + 1.5 × 1.0]
= 2 [2.25 + 1.50 + 1.50]
= 2 [2.2 + 1.5 + 1.5]
= 2 × 5.2 = 10.4 cm^2

3. 3

$$t.u_1\cos\theta_1 = 5 \Rightarrow 5/u_1\cos\theta_1 = t$$

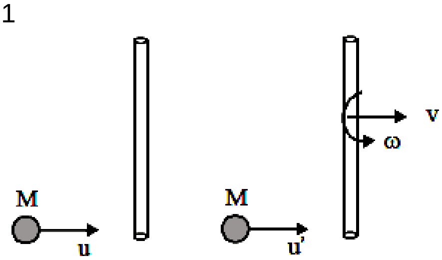
$$t.u_2\cos\theta_2 = 10; t = \frac{10}{u_2\cos\theta_2}$$

Now

$$F = \frac{\Delta P}{\Delta t} = \frac{2m\left[\frac{5}{t} + \frac{10}{t}\right]}{2t} = \frac{15m}{t^2} = \frac{15 \times m}{2h} \times g$$

$$F = \frac{15 \times 0.2 \times 10}{2 \times 2} = 7.5N$$

4. 1



Momentum conservation
Mu = Mv + Mu
u = v + u
Angular momentum conservation

$$M.u\frac{\ell}{2} = Mu'\frac{\ell}{2} + \frac{M\ell^2}{12}.\omega$$

$$u - u' = \frac{\omega\ell}{6}$$

For elastic collision $v + \frac{\omega\ell}{2} - u' = u$

On solving $v = \frac{2u}{5}; \omega = \frac{12u}{5\ell}$

Now KE of upper half part

$$= \frac{1}{2}\frac{M}{2}.\left[\frac{2u}{5} - \frac{3u}{5}\right]^2 + \frac{1}{2}\frac{\frac{M}{2}.\left(\frac{\ell}{2}\right)^2}{12}\left(\frac{12u}{5\ell}\right)^2$$

$$= \frac{Mu^2}{25}$$

5. 3

For the given situation disc will perform translator motion in radius ℓ. Hence case is like simple pendulum (Refer to H.C.V. exercise SHM)

6. 4

For speed to be zero

$$\frac{1}{2}K(S-\ell)^2 = mg(S)$$

$$S^2 + \ell^2 - 2S\ell = \frac{2mg}{K}.S$$

$$\Rightarrow KS^2 - 2\ell KS - 2mgS + K\ell^2 = 0$$

$$S = \frac{(2\ell K + 2mg) \pm \sqrt{4(\ell K + mg)^2 - 4K\ell^2 \times k}}{2K}$$

$$S = \frac{(K\ell + mg) \pm \sqrt{K^2\ell^2 + m^2g^2 + 2\ell Kmg - K^2\ell^2}}{K}$$

$$S = \frac{(K\ell + mg) + \sqrt{2mgK\ell + m^2g^2}}{K}$$

Now maximum speed will be at equilibrium position

$$\frac{1}{2}mv^2 + \frac{1}{2}K\left[\frac{mg}{K}\right]^2 = mg\left(\ell + \frac{mg}{K}\right)$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\frac{m^2g^2}{K} = mg\ell\frac{m^2g^2}{K}$$

$$\frac{1}{2}mv^2 = mg\ell + \frac{m^2g^2}{2K}$$

$$v = \sqrt{2g\ell + \frac{mg^2}{K}}$$

Time of free fall is $\sqrt{\frac{2\ell}{g}}$

And clearly option 4 is wrong.

7. 1

Diameter = 1 $\times$ pitch + L.C $\times$ C.S.R

= 1.5 mm + $\frac{1.5\text{mm}}{100} \times 76$ = 2.64 mm

8. 4

Particle speed $= -v_w \times \text{slope}$

$$= 2 \times (2/1) \text{at} \frac{3}{4} \text{sec}.$$

9. 4

$$C = \frac{Q}{n\Delta T}\frac{\Delta U + W}{n\Delta T} = \frac{nC_v\Delta t + \int_{V_i}^{V_f} PdV}{n\Delta T}$$

$$= C_v + \frac{\alpha}{n\Delta t}.\int_{V_i}^{V_f} VdV$$

$$= \frac{R}{\gamma - 1} + \frac{1}{2}\frac{\alpha V_f^2 - \alpha V_i^2}{n\Delta T}$$

$$= \frac{R}{\gamma - 1} + \frac{1}{2}\frac{P_fV_f - P_iV_i}{n\Delta T}$$

$$= R\left[\frac{1}{\gamma - 1} + \frac{1}{2}\right] = \frac{R}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right)$$

10. 2

At some distance from centre inside core

$$F = -\left(\frac{G\frac{4}{3}\pi r^3(3\rho)m}{r^2}\right)$$

$$ma = -4\pi G\rho mr$$

$$a = -4\pi G\rho r$$

So $\omega = \sqrt{4\pi G\rho} = \frac{2\pi}{T}$

or $T = 2\pi.\sqrt{\frac{1}{4\pi G\rho}} = \sqrt{\frac{\pi}{G\rho}}$

Now time for A to B $\frac{1}{2}\sqrt{\frac{\pi}{G\rho}}$

11. 4

For energy in radius $r_i = \left|-\frac{GMm}{2r}\right| = nk$

Where n is integer k is constant energy, now $\frac{GMm}{2R} = nk$

$$\frac{GMm}{2\left(\frac{3R}{2}\right)} = (n-1)k$$

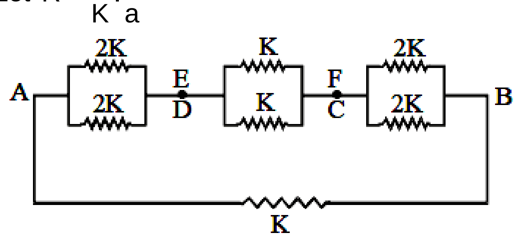
So solving $k = \frac{GMm}{6R}$

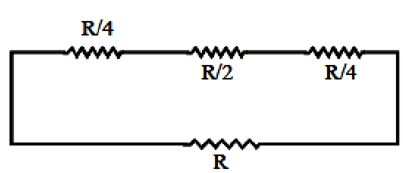
Now this implies that for R_{max}

$$\frac{GMm}{2R_{max}} = (1)\frac{GMm}{6R} \Rightarrow R_{max} = 3R$$

12. 4

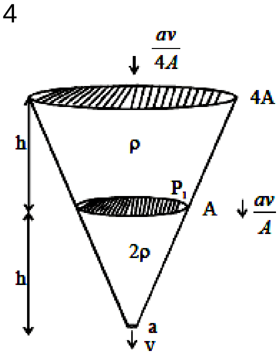
Let $R = \frac{1}{K}.\frac{\ell}{a}$





$$R_{eq} = \frac{R}{2} = \frac{1}{2}\left(\frac{\ell}{ka}\right)$$

13. 4



$$P_0 + \frac{1}{2}2\rho v^2 = P_1 + 2\rho gh + \frac{1}{2}(2\rho)\left(\frac{av}{A}\right)^2$$

$$P_0 + \frac{1}{2}\rho\left(\frac{av}{4A}\right)^2 + \rho gh = P_1 + O + \frac{1}{2}\rho\left(\frac{av}{A}\right)^2$$

$$\Rightarrow \rho v^2 - \frac{\rho}{32}\frac{a^2v^2}{A^2} + \rho gh = 2\rho gh + \rho\frac{a^2v^2}{A^2} - \frac{\rho}{2}\frac{a^2v^2}{A^2}$$

$$v^2\left[1 - \frac{a^2}{32A^2} - \frac{a^2}{A^2}.v^2 + \frac{a^2v^2}{2A^2}\right] = gh$$

$$v = \sqrt{\frac{3gh}{1 - \frac{17a^2}{32A^2}}}$$

14. 2

$$F = (P_0 - P_{average}).\ell h$$

$$F = \left[\frac{2T}{d} - \frac{\rho gh}{2}\right]\ell h$$

$$h = \frac{2T}{\rho gd} \text{ so } F = \frac{2T^2\ell}{\rho gd^2}$$

15. 3

$$\text{Half time} = \frac{\ln(2)}{\text{rate constant}} = \frac{\ell n(2)}{R/\Delta T_0} = \frac{\ell n(2)\Delta T_0}{R}$$

16. 3

Current in bulbs $B_1 = 1A$

Max current in $B_2 = 2A$

For same illumination current should be same in both bulbs

i.e. $i = I(1 - e^{-t/\tau})$

$I = 2(1 - e^{-t/\tau}) \Rightarrow e^{-t/\tau} = 2$

$$t = \tau \ln 2 = \frac{L}{R}\ln 2 = \frac{20}{2}\ln 2 = 7 \text{ seconds}$$

Also, angle rotated by wheel in given time,

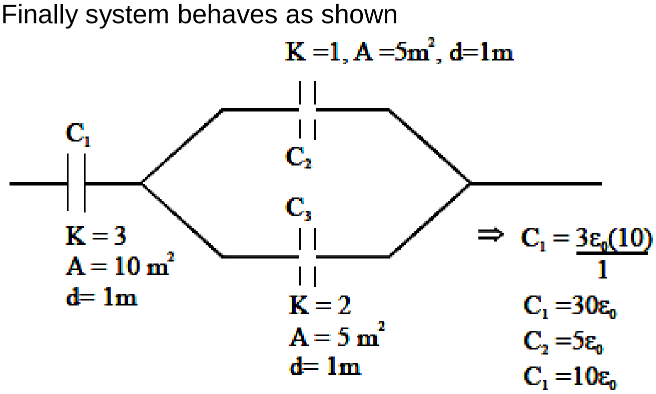
$$\theta = \omega_i t - \frac{1}{2}\propto t^2$$

$$= 2.5\pi(7) - \frac{1}{2}(2)(7)^2 = 6\text{rad} = 343.7°$$

i.e. wheel is 16.3° short of complete revolution at that instant, so, desired colour is green.

17. 1

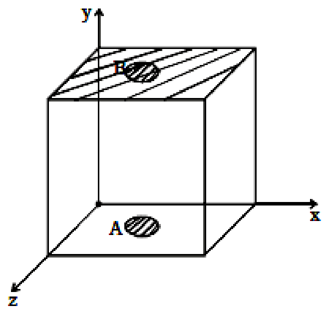
Finally system behaves as shown



$$(\text{net} = \frac{(15\varepsilon_0)(30\varepsilon_0}{45\varepsilon_0} = 10\varepsilon_0 = 8.85 \times 10^{-11} F)$$

18. 4

$V = y^3 + 2$



$\Rightarrow E = \frac{-\delta V}{\delta y}\hat{i} = -3y^2\hat{j}$

$V_A = 2\text{volt}$

$V_B = 10\text{ volt}[V = y^3 + 2]$

$q(V_B - V_A) = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}(8) = \frac{1}{2}(2)V^2$

$\Rightarrow V = 2\text{ m/s}$

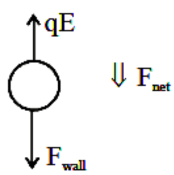
So, velocity of ball before collision = (2 m/s) $\hat{j}$

So, velocity of ball after collision $= -(1.5\text{ m/s})\hat{j}$

Change in momentum $= m(\vec{V}_F - \vec{V}_i) = (-7N.S)\hat{j}$

Net force = (–7) / (0.1) = (–70 N) $\hat{j}$

From FBD of ball during collision



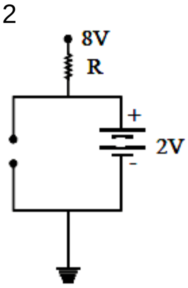
$F_{net} = F_{wall} - qE$

$F_{wall} = F_{net} + qE$

= (70 + 6) = 76 N

[E at top face = $3y^2 = 3(2)^2 = 12N/C$]

19. 2

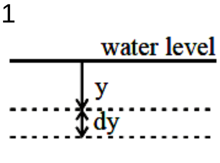


$i = 20mA = \frac{E - V_{LED}}{R} = \frac{8-2}{R}$

$R = \frac{6V}{20mA} = 300\Omega$

Also reverse voltage across red diode is 2V which is fine for LED with reverse breakdown voltage of 3V.

20. 1



$$h_{app} = \mu_{observer}\sum\left(\frac{h_i}{\mu_i}\right) = \int_0^1 \frac{dy}{y^2+1} = \tan^{-1}(1) = \frac{\pi}{4}$$

21. 4

X	Y	Z	$\overline{X.Y}$	$\overline{Y.Z}$	$R = \overline{\overline{X.Y} + \overline{Y.Z}}$
0	0	0	1	1	0
0	1	0	1	1	0
0	0	1	1	1	0
0	1	1	1	0	0
1	0	0	1	1	0
1	1	0	0	1	0
1	0	1	1	1	0
1	1	1	0	0	1

From truth table, answer is AND gate.

22. 4

$$B = \frac{E}{C} = \frac{10^{-4}}{3\times10^{8}} = 3.3\times10^{-13}\,T$$

23. 3

Let E_c be the amplitude of carrier wave and Es is signal amplitude

then $E_C = \left(\frac{V_{max}+V_{min}}{2}\right), E_s = \left(\frac{V_{max}-V_{min}}{2}\right)$

$$E_C = mE_C \Rightarrow 4 = M6 \Rightarrow M = \frac{2}{3}$$

24. 1

$$\varepsilon - \left(\vec{v}\times\vec{B}\right).\vec{\ell}_{eff}$$

$$\varepsilon = \left[2\hat{i}\times\left(3\hat{j}+4\hat{k}\right)\right].\left[3\hat{i}+4\hat{j}\right]$$

$$|\varepsilon| = 32 \text{ volt}$$

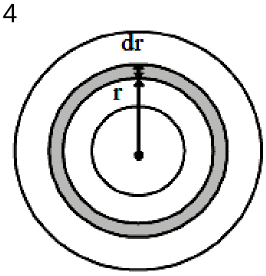
25. 2

$$\frac{X_i}{X_f} = \left|\frac{\frac{1}{\omega C_i} - \omega L_i}{\frac{1}{\omega C_F} - \omega L_f}\right|$$

$$= \frac{\frac{\ell}{\omega K\varepsilon_0 A} - \omega\mu_0 n^2 A\ell}{\frac{\ell}{\omega\varepsilon_0 A} - \omega\mu_0\mu_r n^2 A\ell} = \frac{1}{K}\left(\frac{1-K}{1-\mu_f}\right)$$

[Using $\varepsilon_0\mu_0 = \frac{1}{C^2}$] & $[\omega^2 A^2 n^2 = C^2]$

26. 4



$$dR = \frac{\rho dr}{4\pi r^2}$$

$$R = \int dR = \frac{\rho}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} \Rightarrow R = \frac{\rho(r_2 - r_1)}{4\pi r_1 r_2}$$

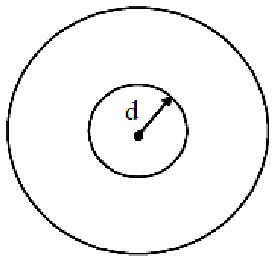
Rate of melting is max when power dissipated in sphere is max. Using maximum power transfer theorem,

R = r, of batter i.e. $\frac{\rho(r_2 - r_1)}{4\pi r_1 r_2} = \frac{2}{\pi}$

$\rho = \frac{8r_1r_2}{r_2 - r_1} = \frac{8(200)}{10} = \frac{160}{100}$(in SI)

Also, $\sigma = \frac{1}{\rho} = \frac{10}{16} = \frac{5}{8}$

27. 2



Considering Gaussian surface

$\phi = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\, 4\pi d^2 = \frac{2ed^3}{\varepsilon_0 R^3}$

$E = \frac{2edk}{R^3}$

For equilibrium of charge,

$\frac{kee}{(2d)^2} = \frac{2d}{R^3} \Rightarrow d^3 = \frac{R^3}{8} \Rightarrow d = R/2$

28. 2

When light enters medium of refractive index μ, its speed decreases, to $\frac{c}{\mu}$.

∴ Wavefront at point P is option (2).

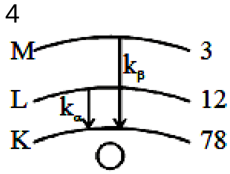
29. 2

$y_{n^{th}} = \frac{n\lambda_1 D}{2d}; n = 2,4,6,8.....$

$y_{m^{th}} = \frac{n\lambda_2 D}{2d}; m = 2,4,6,8.....$

$y_{n^{th}} = \lambda_{m^{th}} = \frac{d}{2}$ from central fringe.

30. 4



$$\frac{\left(\frac{hc}{\lambda_{k_\alpha}}\right)}{\left(\frac{hc}{\lambda_{k_\beta}}\right)} = \frac{(E_k - E_L)}{(E_k - E_m)}$$

$$\Rightarrow \frac{\lambda_{k_\beta}}{\lambda_{k_\alpha}} = \frac{78-12}{78-3} = \frac{22}{25} = \frac{\lambda_{k_\alpha}}{\lambda_{k_\beta}} = \frac{25}{22}$$

CHEMISTRY

31. 1

$$\left[12\times\frac{98.8}{100}\right]+\left[13.1\times\frac{1.18}{100}\right]+\left[14.1\times\frac{0.02}{100}\right]$$
$$+\left[4\times1.008\right]$$
$$\left(\because \text{for } {}^1_1H\right)$$
$= 16.0454$

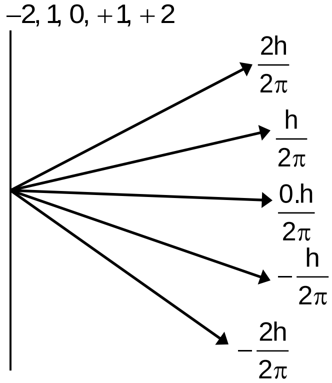
32. 4

Quantum number 'm' refers to the projection of angular momentum in the arbitrarily choosen direction and it specified the orientation, along quantization axis

$$L = m\times\frac{h}{2\pi}$$

e.g $\ell = 2$ (d orbitals)

$m = -2, 1, 0, +1, +2$



33. 4

T_C α Ease of liquification
α Vander wall's constant 'a'
α Force of attraction

$V_c(H_2O) > V_c(NH_3)$

Density (H_2O) <Density $(NH_3) = 0.0445$

= 0.003155

34. 4

Fact

35. 2

Molar enthalpy of formation for any substance in its standard state of aggregation is taken as zero.

36. 2

$$\log^{\beta}\underset{(\text{Complex})}{(NH_3)} < \log^{\beta}\underset{(\text{Complex})}{(CN)}$$

$\Rightarrow$ Common ion decrease the solubility

$\Rightarrow$ Strength of $Ca(CN)_2$ i.e. bond strength is higher than KCN (more ionic)

37. 1

Each reactant not necessarily appear in rate law expression

38. 4

$$\left[r^+\right]+\left[r^-\right]=\frac{\sqrt{3}a}{2}$$
$$a=\frac{180\times2}{\sqrt{3}}=\frac{60\times2\times3}{\sqrt{3}}$$
$$a=120\sqrt{3}$$

C $\because$ cation is in body centre and distance between body center in 'a'

39. 4

Factual

40. 2

$$2\left[\wedge_m^0\left(Na^+\right)\right]+\left[\wedge_m^0\left(SO_4^{\ 2-}\right)\right]=\frac{K}{C}$$

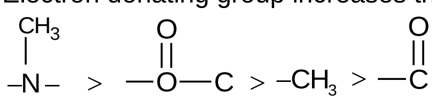
$$\wedge_m^0\left(SO_4^{\ 2-}\right)=\left[\frac{0.26\times10^{-3}}{\underset{\left(mol/cm^3\right)}{0.001\times10^{-3}}}\right]-\left[2\times50\right]$$

$= 260 - 100$

$= 160\ s\ cm^2\ mol^{-1}$

41. 2

Electron donating group increases the reactivity towards EAS

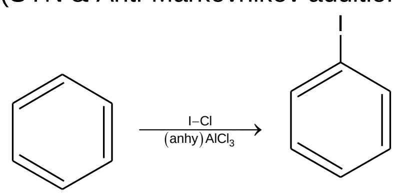


42. 3

Hydroboration oxidation

(SYN & Anti-Markovnikov addition)

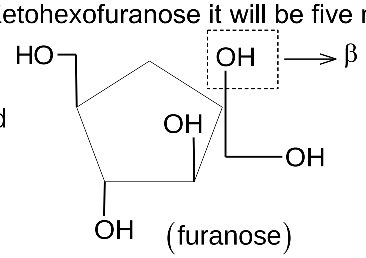
43.



$$I-Cl+AlCl_3\rightarrow I^{+}+AlCl_4^{\ -}$$

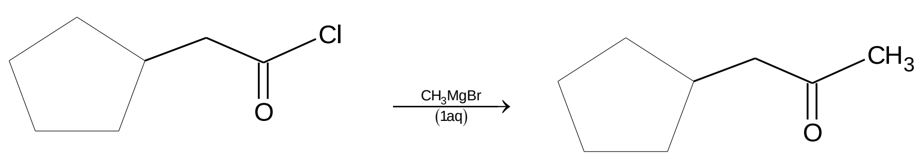
44. 1

Since it is β-Ketohexofuranose it will be five membered ring also 'OH' group. On anomeric carbon will be upward

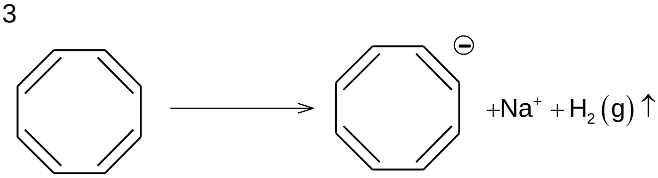


(furanose)

45. 4



46. 3



(highly unstable)

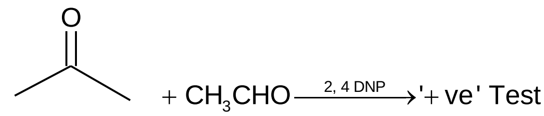
47. 1

$$CH_3CHO\xrightarrow{I_2/NaOH}HCOO-Na^{+}+CHI_3$$

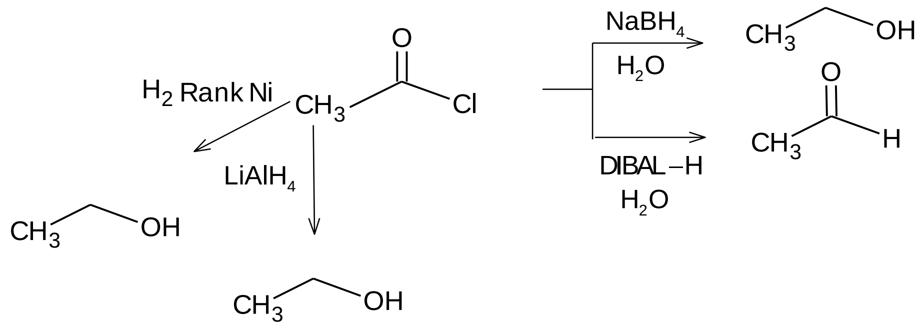
$$HCHO\xrightarrow{I_2/NaOH}NO\ reaction$$

$PhCHO\ \&\ CH_3CHO\rightarrow$ '+ve' Tollen test

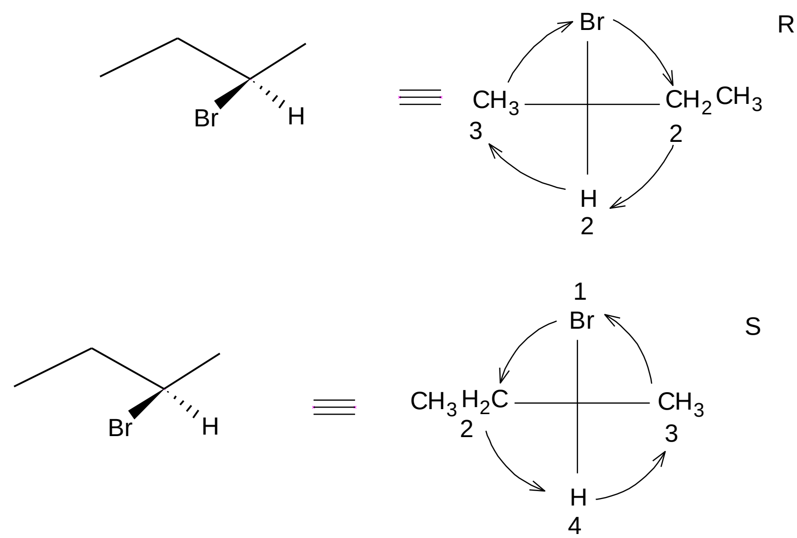
Glucose & Fructose $\xrightarrow{Br_2-H_2O}$ No decolourisation (give acid)


O
+ CH3CHO
2, 4 DNP
'+ ve' Test

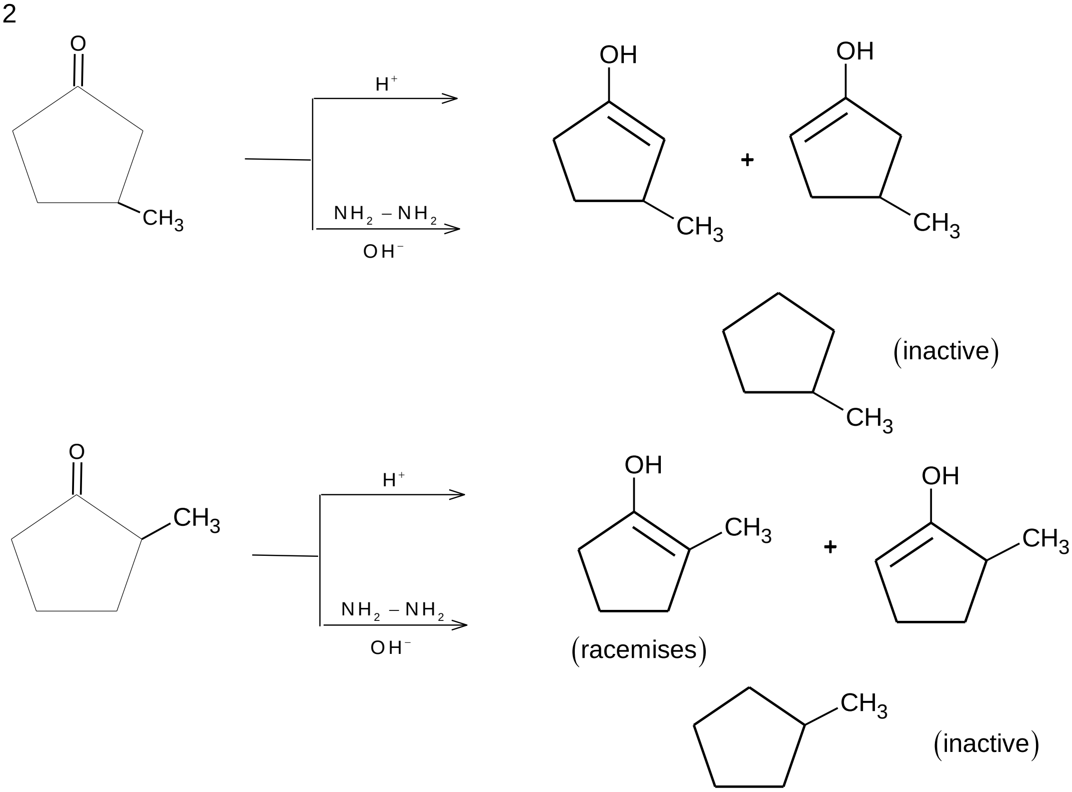
48. 4


NaBH4
H2O
CH3 OH
O
H2 Rank Ni
CH3
Cl
O
CH3
H
DIBAL – H
H2O
LiAlH4
CH3 OH
CH3 OH

49. 2


Br
H
Br
R
CH3
3
CH2 CH3
2
H
2
Br
H
1
Br
S
CH3 H2C
2
CH3
3
H
4

50. 2


O
CH3
H+
NH2 – NH2
OH−
OH
CH3
+
OH
CH3
CH3
(inactive)
O
CH3
H+
NH2 – NH2
OH−
OH
CH3
+
OH
CH3
(racemises)
CH3
(inactive)

51. 2

$Cl_2 > Br_2 > F_2 > I_2$

(Bond energy)

52. 1

$sp^3 - sp^3 > sp^2 - sp^2$

(bond length)

Further $\underset{(\text{diamond})}{C-C} > \underset{(\text{due to large separation})}{C-C}$ (graphite)

53. 2

$Li_2NO_3 \xrightarrow{\Delta} Li_2O + NO$

$KNO_3 \xrightarrow{\Delta} KNO_2 + \frac{1}{2}O_2$

$Zn + (\text{conc.})HNO_3 \longrightarrow Zn(NO_3)_2 + NO_2$

$H_2SO_4 + NaNO_3 \longrightarrow Na_2SO_4 + H_2O + NO_2$

54. 4

Al $\Rightarrow$ electrolytically

Zn $\Rightarrow$carbon reduction

Fe $\Rightarrow$ carbon reduction

Pb $\Rightarrow$ self reduction

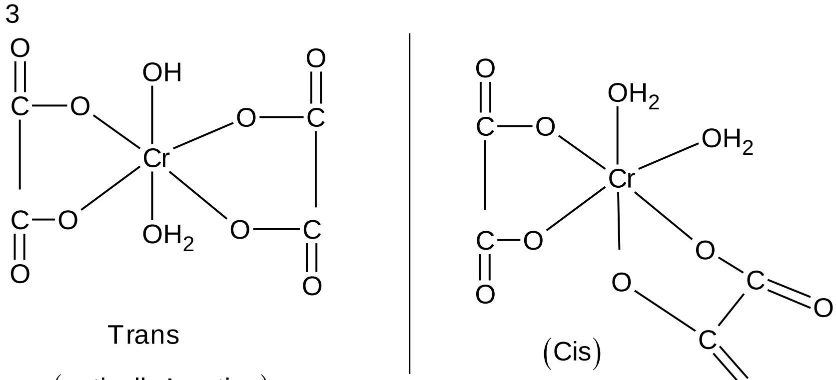
55. 2

$Al \Rightarrow [Cu(NH_3)_4]^{2+} \Rightarrow sp^2d \Rightarrow$ sq. planer

$[Ni(NH_3)_6]^{2+} \Rightarrow sp^3d^2 \Rightarrow$ octahedral

$[Zn(NH_3)_4]^{2+} \Rightarrow sp^3 \Rightarrow$ Tetrahdral

56. 3



Trans (optically Inactive)

(Cis) (opticaly active)

57. 1

$P_4 + NaOH \longrightarrow PH_3 + NaH_2PO_2$

58. 4

$Al_2(SO_4)_3 \xrightarrow{NH_4OH} Al(OH)_3 + NH_4SO_4$

$ZnSO_4 \xrightarrow{NH_4OH} Zn(OH)_2 + NH_4SO_4$

$Al_2(SO_4)_3 \xrightarrow{NaOH} Al(OH)_3 + Na_2SO_4$

$ZnSO_4 \xrightarrow{NaOH} Zn(OH)_2 + Na_2SO_4$

59. 4

$MnO_4^- + I^- \rightarrow MnO_2 + I_2$

$O_3 + I^- \rightarrow O_2 + I_2$

$CuSO_4 + I^- \rightarrow CuI_2 \rightarrow CO_2I_2 + I_2$

60. $MgN_3 + H_2O \rightarrow Mg(OH)_2 + NH_3$

$NaNO_3 + Zn + NaOH \longrightarrow NH_3 + Na_2ZnO_2 + H_2O$

$CaNCN + H_2O \longrightarrow CaCO_3 + H_2O$

MATHEMATICS

61. 2

$\left(x\cot y dy + \ln\sin y dx\right) + \left(\ln\cos x dy - y\tan x dy\right) = 0$

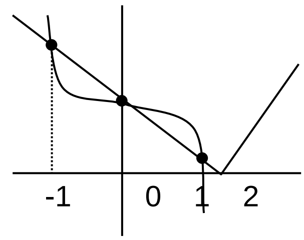
$d\left(x\ln\sin y\right) + d\left(y\ln\cos x\right) = 0$

$x\ln\sin y + y\ln\cos x = \ln c$

$\left(\sin y\right)^x.\left(\cos x\right)^y = c$

62. 4

By graph



63. 3

$f(x) = 2 + \dfrac{3}{x^4 - 7x^2 - 4x + 23}$

$3 \le x^4 - 7x^2 - 4x + 23 < \infty$

$0 < \dfrac{1}{x^4 - 7x^2 - 4x + 23} \le \dfrac{1}{3}$

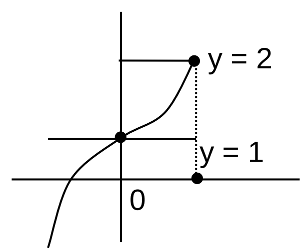
$0 < \dfrac{3}{x^4 - 7x^2 - 4x + 23} \le 1$

$2 < 2 + \dfrac{3}{x^4 - 7x^2 - 4x + 23} \le 3$

64. 2

$\text{ANS} = \int_0^1 \left(2 - \left(x^3 - 3x^2 + 3x + 1\right)\right)dx$

$= \dfrac{1}{4}$



65. 2

Let $f(x) = ax + b$

$f(0) = 1 \Rightarrow b = 1$

$f'(0) = 2 \Rightarrow a = 2$

Period of $\sin(2x+1) = \pi$

66. 4

$\sum_{k=1}^{12} 12.K.{}^{12}C_k.{}^{11}C_{k-1} = 12\times 12\sum_{k=1}^{12}\left({}^{11}C_{k-1}\right)^2$

$= 12\times 12\times {}^{22}C_{11}$

67. 3

$x = 3t + 2$

$y = 2t - 1$

$z = -t + 1$

$\Rightarrow z = 0$

$t = 1$

$(3t + 2)(2t - 1) = c^2$

$c = \pm\sqrt{5}$

68. 2

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Put $x = t - \frac{1}{t}$

$$dx = \left(1 + \frac{1}{t^2}\right)dt$$

$$\int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)\left(1 + \frac{1}{t^2}\right)dt = 1$$

$$\int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)dt + \int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)\left(\frac{1}{t^2}\right)dt = 1$$

Put $t = \frac{1}{z}$

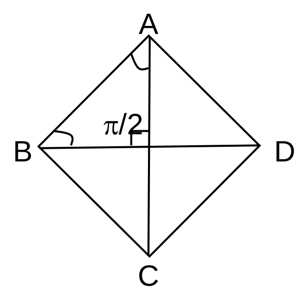
$$dt = -\frac{1}{z^2}dz$$

$$\int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)dt + \int_{-\infty}^{0} f\left(t - \frac{1}{t}\right)\frac{1}{t^2}dt + \int_{0}^{\infty} f\left(t - \frac{1}{t}\right)\frac{1}{t^2}dt = 1$$

$$\int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)dt + \int_{0}^{\infty} f\left(\frac{1}{z} - z\right)z^2\left(-\frac{1}{z^2}\right)dz + \int_{\infty}^{0} f\left(z - \frac{1}{z}\right)z^2\left(-\frac{1}{z^2}\right)dx = 1$$

$$\int_{-\infty}^{\infty} f\left(t - \frac{1}{t}\right)dt = 1$$

69. 1



$BD = 2R_1 \sin A$

$AC = 2R_2 \sin B$

$$Area = \frac{1}{2}\left[4R_1R_2 \sin^2 A\right]$$

$$\tan A / 2 = \frac{1}{2}$$

$$\sin A = \frac{4}{5}$$

$$Area = \frac{1}{2}\left[4 \times \frac{25}{2} \times 25 \times \frac{16}{25}\right]$$

$= 400$

70. 4

$-1 \le |z| \le 2$

$\left|z^2 + z\sin\theta\right| \le |z|^2 + |z|\sin\theta \le 4 + 2 \le 6$

71. 3

For symmetry about x = k

a = 0

$y = bx^2 + cx + d$ is symmetric about $x = -\frac{c}{2b} = k$

72. 3

$-\left(\tan^{-1}x\right)^2 + \tan^{-1}(x) + 2 > 0$

73. 2

$x \in [2, 4]$

$f'(x) = 2x + 2 \ge 0$

$$a = \int_0^2 \left(t^2 + 2t + 2\right)dt = \frac{32}{3}$$

$b = \int_0^4 \left(t^2 + 2t + 2\right) dt = \frac{136}{3}$

$a + b = 56$

74. 3

$\left(x + 2y + z - 1\right) + \lambda\left(2x + y + 3z - 2\right) = 0$

$x(1 + 2\lambda) + y(2 + \lambda) + z(1 + 3\lambda) - 1 - 2\lambda = 0$

$(1 + 2\lambda).1 + (2 + \lambda).1 + (1 + 3\lambda).1 = 0$

$6\lambda + 4 = 0 \Rightarrow \lambda = -2/3$

So plane $x - 4y + 3z - 1 = 0$

This plane is perpendicular to $x + ky + 3z - 1 = 0$

So, $(1.1) + (-4)(k) + 3 \times 3 = 0$

$k = \frac{5}{2}$

75. 3

$f''(x) = -x^{p-4} \sin\left(\frac{1}{x}\right) - (p-2)\cos\left(\frac{1}{x}\right)x^{p-3}$

$-p\cos\left(\frac{1}{x}\right)x^{p-3} + p(p-1)\sin\left(\frac{1}{x}\right)x^{p-2} + 9.\frac{|x^3|}{x} : x \neq 0$

$= 0$

$\lim_{x \to 0} f''(x) = 0$

So p > 4

76. 4

$\lim_{x \to \frac{1}{2}} \frac{ax^2 + bx + c}{(2x-1)^2} = \frac{1}{2}$

$\frac{a}{4} + \frac{b}{2} + c = 0$

$a = 2$

$a + b = 0$

$b = -2$

$c = 1/2$

77. 4

$\frac{2x^3 + 3x^2 + x - 2}{x^2 + x - 2} = (2x + 1) + \frac{1}{(x-1)} + \frac{3}{(x+2)}$

$\frac{d}{dx}\left(\frac{2x^3 + 3x^2 + x - 2}{x^2 + x - 2}\right) = 2 - \frac{1}{(x-1)^2} - \frac{3}{(x+2)^2}$

$A = 2, B = -1, C = -3$

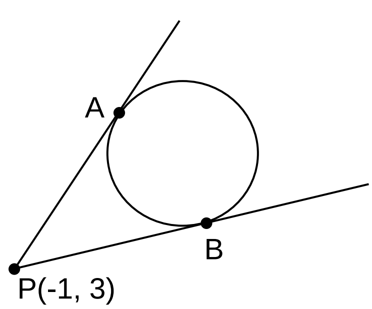
78. 2

$PA = PB = \sqrt{S_1}$

$= \sqrt{1 + 9 + 2 + 12 - 8}$

$= 4$

$PA + PB = 8$



79. 2

$t_n = \left[(n+1)(n+1)! - n \times n!\right] - \left((n+1)! - n!\right)$

$s_{10} = 10 \times 11!$

$t_{10} = 101 \times 10!$

$\frac{t_{10}}{s_{10}} = \frac{101\times 10!}{10\times 11!} = \frac{101}{110}$

80. 2

$g'(x) = (12x^2 - 12x)\left[f'(2x^3 - 3x^2) - f'(6x^2 - 4x^3 - 3)\right] > 0$

Case-I

$12x(x-1) > 0 \;\; \& \;\; f'(2x^3 - 3x^2) \geq f'(6x^2 - 4x^3 - 3)$

$2x^3 - 3x^2 > 6x^2 - 4x^3 - 3$

$x \in \left(-\frac{1}{2}, 0\right)$

Case-II

$12\times(x-1) < 0 \;\; \& \;\; (x-1)^2(2x+1) < 0$

81. 2

Let $f(x) = \frac{\cos x}{x}$

$f'(x) = -\frac{x\sin x - \cos x}{x^2} < 0$

$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

$\frac{\sqrt{3}}{2} < I < \infty$

$0 < J < 1/4$

82. 4

$(x-1)^2 + (y+2)^2 = \gamma$

$x^2 + y^2 - 2x + 4y + 5 - \gamma = 0$

$g_1 = -1 : f_1 = 2 : c_1 = 5 - \gamma$

$g_2 = -2 : f_2 = -2 : c_2 = -1$

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2(-1)(-2) + 2(2)(-2) = 5 - \gamma - 1$

$\Rightarrow 4 - 8 = 4 - \gamma$

$\Rightarrow \gamma = 8, \alpha = 1, \beta = -2$

83. 3

Characteristics equations

$\lambda^3 - 5\lambda^2 + 6\lambda - 1 = 0$

$A^3 = 5A^2 - 6A + I$

$A^3 = (A - I)(5A - I)$

84. 2

p	q	p ^ q	P ^ q → p	-q	q ^ ~ q	(p^q→ p)→(q ^~q)
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

85. 4

$n_1 = 10; \bar{x}_1 = 60 : \sigma_1 = 4$

$n_2 = 10; \bar{x}_2 = 40 : \sigma_2 = 6$

$\sigma^2 = \frac{n_1(\sigma_1^2 + \alpha_1^2) + n_2(\sigma_2^2 + d_2^2)}{20}$

$= \frac{116+136}{2} = 126$

$\sigma = \sqrt{126} = 11.2$

86. 3

$x_1 \boxed{5}\ x_2 \boxed{3}\ x_3 \boxed{2}\ x_4$

$x_1 + x_2 + x_3 + x_4 = 5$

Number of non negative integral solution $= {}^8C_5$

Now they are arrange by 3! way's and distribute among three person = 3!

So, total number of way's $= {}^8C_5 \times 3! \times 3!$

87. 3

$\left[\bar{a}\bar{b}\bar{c}\right] = \bar{a}.(\bar{b}\times\bar{c})$

$= \bar{a}.(2\hat{i}+2\hat{j}-\hat{k})$ is max when $\bar{a}$ & $\bar{b}\times\bar{c}$ collinear

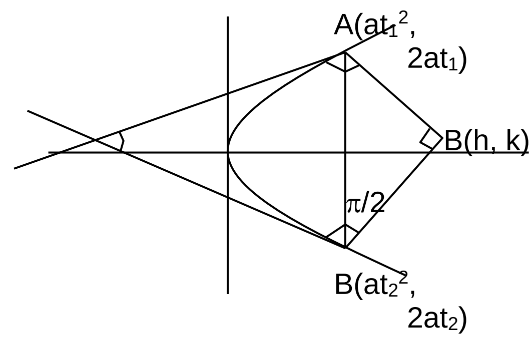
$\bar{a} = \lambda(2\hat{i}+2\hat{j}-\hat{k})$

$|\bar{a}| = |\lambda|(3)$

$\lambda = \frac{1}{3}$

So, $\bar{a} = \frac{1}{3}(2\hat{i}+2\hat{j}-\hat{k})$

88. 1



$t_1t_2 = -1$

$h = 2a + a\left[t_1^2 + t_2^2 + t_1t_2\right]$

$k = -at_1t_2(t_1+t_2)$

$t_1 + t_2 = \frac{k}{a}$ ..(1)

$h = 2a + a.\left[(t_1+t_2)^2 - t_1t_2\right]$

$(h-3a) = a.\frac{k^2}{a^2} \Rightarrow y^2 = a(x-3a)$

89. 2

$n(s) = \frac{12!}{(2!)^6(6!)}$

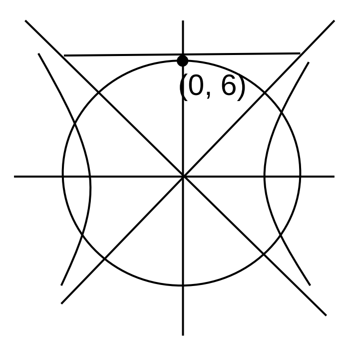
(i) if P_1 and P_2 is in same group

$n(E) = \frac{10!}{(2!)^5(5!)}$

$P(E_1) = \frac{1}{11}$

$= \frac{1}{11}\times 1 + \left(\frac{10}{11}.\frac{1}{2}.\frac{1}{2}\right)2 = \frac{6}{11}$

90. 2



$P(\alpha,\beta)$

Asymptotes $y = \pm\frac{4}{5}x$

$P(\alpha,\beta)$ out side circle