FIITJEE

FULL TEST – 2
JEE MAIN

DATE: 04/01/2019

ANSWERS KEY

PHYSICS

1.	1	9.	3	17.	2	25.	2
2.	4	10.	1	18.	1	26.	3
3.	2	11.	1	19.	2	27.	3
4.	1	12.	1	20.	4	28.	2
5.	4	13.	2	21.	4	29.	3
6.	3	14.	4	22.	1	30.	4
7.	1	15.	4	23.	3		
8.	1	16.	1	24.	3		

CHEMISTRY

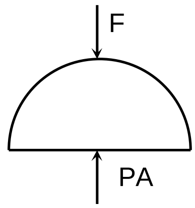
31.	**4**	39.	1	47.	3	55.	3
32.	**1**	40.	1	48.	3	56.	4
33.	4	41.	2	49.	3	57.	1
34.	2	42.	3	50.	4	58.	2
35.	4	43.	1	51.	4	59.	4
36.	3	44.	1	52.	2	60.	4
37.	1	45.	3	53	3		
38.	3	46.	3	54.	4		

MATHEMATICS

61.	1	69.	3	77.	4	85.	3
62.	3	70.	3	78.	3	86.	2
63.	3	71.	1	79.	1	87.	1
64.	4	72.	3	80.	3	88.	3
65.	2	73.	1	81.	2	89.	1
66.	3	74.	3	82.	1	90.	2
67.	1	75.	4	83.	1		
68.	1	76.	4	84.	2		

HINTS & SOLUTIONS
PHYSICS

1. 1



$$PA - F = \frac{2}{3}\pi r^3 \rho_1 g$$

$$(P_0 + h\rho_1 g)\pi r^2 - F = \frac{2}{3}\pi r^3 \rho_1 g$$

$$F = P_0 \pi r^2 + \left(h - \frac{2}{3}r\right) r^2 \rho_1 g$$

2. 4

P = Constant

$$P_1 = \frac{V_1^2}{Z_1}$$

$$P_2 = \frac{V_2^2}{Z_2}$$

$$\frac{V_1^2}{Z_1} = \frac{V_2^2}{Z_2}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{Z_1}{Z_2}}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

So $\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$

3. 2

Work done by all forces = Δk

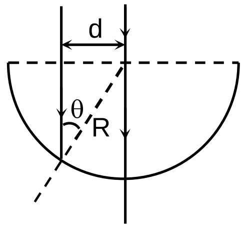
$$\int_a^b -\frac{GMm}{r^{2.1}} dr = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$-GMm\left[-\frac{1}{1.1 r^{1.1}}\right]_a^b = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\frac{GMm}{1.1}\left[\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right] = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$v = \sqrt{\frac{2GMm}{1.1}\left[\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right] + u^2}$$

4. 1



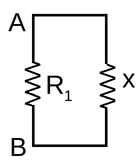
No light will exist at lower surface, if there is total internal reflection at lower surface

$$\mu \sin\theta = 1 \sin 90°$$

$$\sin\theta = \frac{1}{\mu}$$

$$\mu = \frac{1}{\sin\theta} = \frac{1}{d/R} = \frac{R}{d}$$

5. 4



$$\frac{R_1 x}{R_1 + x} = R_{AB}$$

$$\frac{4x}{4 + x} = 2$$

4x = 8 + 2x

2x = 8

x = 4

Now $\frac{6 \times 4}{6 + 4} = R'_{AB}$

$$R'_{AB} = \frac{24}{10} = 2.4$$

6. 3

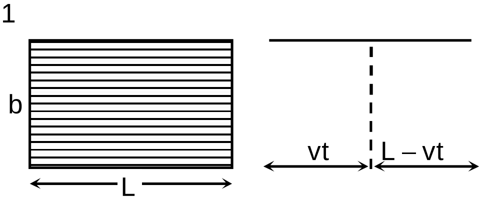
Conceptual

(Smaller the least count of instrument, more will be precision)

7. 1

If we increase angle of liquid wedge, path difference between the rays reaching at microscope increases and hence fringe width will decrease.

8. 1



$$C = \frac{\epsilon_0 b}{d}\left[vt + k(L - vt)\right]$$

$$= \frac{\epsilon_0 b}{d}\left[kL - vt(k - 1)\right]$$

So graph of C Vs t should be linear.

9. 3

$$\varepsilon = \frac{d\phi}{dt} = Bvd$$

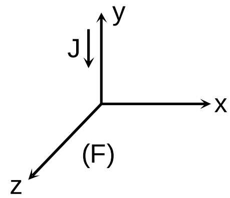
$$R_{net} = R + r.\frac{d}{\sin\theta}$$

$$I = \frac{\varepsilon}{R_{net}}$$

$$F = BI\left(\frac{d}{\sin\theta}\right) = B.\frac{Bvd}{\left(R + r\frac{d}{\sin\theta}\right)}.\frac{d}{\sin\theta} = \frac{\frac{B^2vd^2}{\sin\theta}}{\left(h + r\frac{d}{\sin\theta}\right)}$$

10. 1

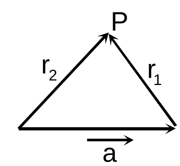
$$P = \frac{F}{A} = \frac{BIh}{A} = \frac{B(JwL)h}{hw} = JLB$$



F will be along –ve z-axis

11. 1

Magnetic field inside the hollow tube depends only on distance between centre of two spheres of independent of position of paint. Net magnetic field should be directed along line joining the centre.



$\therefore \qquad B = \frac{\mu_0 Ja}{2}$ along x-axis

12. 1

$I = I_0 \cos^2\theta$

Intensity of unpolarised light after passing through polarises becomes half.

$$I = \frac{I_0}{2}\cos^2\omega t.\cos^2(90 - \omega t)$$

$$= \frac{I_0}{2}\cos^2\omega t \sin^2\omega t$$

$$= \frac{I_0}{8}\sin^2 2\omega t$$

$$= \frac{I_0}{16}[1 - \cos 4 cot]$$

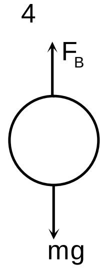
13. 2

$$I = \frac{1}{2}\epsilon_0 E_{max}^2 c$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$I = \frac{1}{2}\frac{\epsilon_0 E_{max}^2}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{max}^2$$

14. 4



$$T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{\rho_{He}}{(\rho_{air} - \rho_{He})}}$$

$$g_{eff} = \frac{(F_B - mg)}{m} = \frac{V\rho_{air}g - V\rho_{He}g}{V\rho_{He}} = \left(\frac{\rho_{air} - \rho_{He}}{\rho_{He}}\right)g$$

15. 4

Let mass of each astronaut = m

Conserving angular momentum

$$I\omega = I'\omega'$$

$$[Nm]r^2\omega - [N-1]r^2\omega'$$

$$\omega' = \left(\frac{N}{N-1}\right)\omega$$

Initial force of any astronaut is

$$F = mr\omega^2$$

So $g = r\omega^2$

$$g' = r\omega'^2$$

$$\frac{g}{g'} = \left(\frac{\omega}{\omega'}\right)^2 = \left(\frac{N-1}{N}\right)^2$$

$$\frac{g'}{g} = \left(\frac{N}{N-1}\right)^2$$

16. 1

$$f = \frac{1}{2d}\sqrt{\frac{T}{\mu}}$$

$$f^2 = \frac{1}{4d^2}\frac{T}{\mu}$$

$$T = 4\mu d^2 f^2$$

17. 2

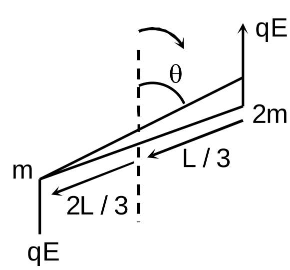
Concept

Mass of muon is 207 times more than $\vec{e}$.

Energy will be $E_\mu = 200E_e$

18. 1

$$I = m\left(\frac{2L}{3}\right)^2 + 2m\left(\frac{L}{3}\right)^2$$



$$= \frac{4mL^2}{9} + \frac{2mL^2}{9}$$

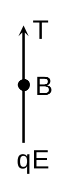
$$= \frac{6mL^2}{9} = \frac{2}{3}mL^2$$

$$\tau = qEL\sin\theta = qE\theta$$

$$\tau = I\alpha$$

$$qEL\theta = \frac{2mL^2}{3}\alpha$$

$$\alpha = \frac{3qE}{2mL}\theta$$



$$\omega^2\theta = \frac{3qE}{2mL}\theta$$

$$\omega_f^2 = \frac{3qE}{2mL}$$

$$T - qE = mr(\omega_f\theta)^2$$

$$T - qE = m\left(\frac{2L}{3}\right)\frac{3qE}{2mL}\theta_0^2$$

$T - qE = qE\theta_0^2$

$T - qE + qE\theta_0^2$

19. 2

In the tunnel, only diffracted signal with reach and for diffraction, size of obstacle must be of the order of wavelength $v = \nu\lambda \Rightarrow \lambda = \frac{v}{\nu}$

Here $v = c =$ speed of light

$$\lambda_1 = \frac{3\times10^8}{3\times10^6} = 10^2 = 100\text{m}$$

$$\lambda_2 = \frac{3\times10^8}{30\times10^6} = 10\text{m}$$

$$\lambda_3 = \frac{3\times10^8}{3\times10^9} = 0.1\text{m}$$

So, $\lambda_3 = d$

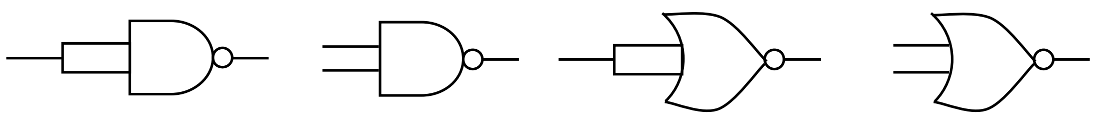
So, signals of frequency 30 MHz will reach to tunnel.

20. 4

Glass slab will produce shift in the direction of incident ray. So, for lens, object will appear between P and Q, hence image will form further away from S.

If converging lens is placed, then it will converge the ray further reaching to S and hence image will form between R & S.

21. 4



Now make calculation based on input signal.

22. 1

H = ni

B = μH

On heating the care beyond curie temperature T_C, ferromagnetic substance changes to paramagnetic.

So, μ will decrease

So, magnetic field will decrease. H will remains same.

23. 3

Frequency will change only when there is a relative motion between source and observer, so frequency will remains same.

Due to velocity of wind, velocity of sound increases

i.e. $340 + 10 = 350$ m/s

$v' = \nu\lambda$

$$\lambda' = \frac{350}{400} = \frac{5}{8} = 0.625$$

$$v = \nu\lambda,\ \lambda = \frac{340}{400} = \frac{17}{20} = 0.85$$

So wave length will decrease.

24. 3

$$\varepsilon_1 = 1 - \frac{Q_m}{Q_b}$$

$$\varepsilon_2 = 1 - \frac{Q_c}{Q_m}$$

$$\varepsilon_{net} = 1 - \frac{Q_c}{Q_b}$$

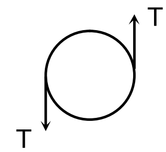
Solving,

$$\varepsilon_{net} = \varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2$$

$$\text{Efficiency} = \frac{\text{Work done}}{\text{Heat absorbed}} = \frac{\text{Heat absorbed} - \text{Heat released}}{\text{Heat absorbed}} = 1 - \frac{\text{Heat released}}{\text{Heat absorbed}}$$

25. 2

Angular impulse $= \int T(2R)dt$



By impulse momentum theorem

$2TRt = L_f - L_i = I\omega = mR^2\omega$

Now centripetal force = mg = $mR\omega^2$

$$\omega = \sqrt{\frac{g}{R}}$$

So, $2TRt = mR^2\sqrt{\frac{g}{R}}$

$$t = \frac{m\sqrt{Rg}}{2T}$$

26. 3

Applying conservation of angular momentum

$$I_P\left(\frac{v_{rel}}{R}\right) = (I_P + I_E)\omega$$

$$\omega = \frac{I_P}{(I_P + I_E)}\frac{v_{rel}}{R}$$

Earth moves west to east.

So if people run due west, the change in angular velocity of earth will be

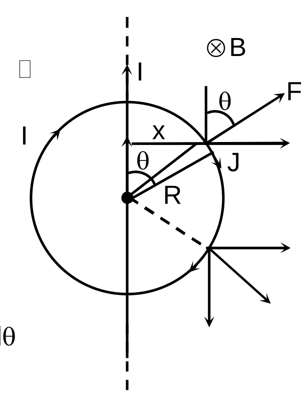
$$\omega_0 - \frac{I_P v_{rel}}{(I_P + I_E)R}$$

27. 3

Sharpness of V v_s ω/ω_0 curve decide the quality factor. More sharp is curve, more will be quality factor.

28. 2

Force on small element



$$dF = IdLB$$

$$= IRd\theta\frac{\mu_0 I}{2\pi x}$$

$$x = R\sin\theta$$

$$dF = \frac{\mu_0 I^2 Rd\theta}{2\pi R\sin\theta} = \frac{\mu_0 I^2 d\theta}{2\pi\sin\theta}$$

Now vertical component will cancel out

$$dF_H = dF\sin\theta - \frac{\mu_0 I^2}{2\pi}\frac{d\theta}{\sin\theta}.\sin\theta = \frac{\mu_0 I^2}{2\pi}d\theta$$

$$F = \int_0^{2\pi}\frac{\mu_0 I^2}{2\pi}d\theta = \mu_0 I^2$$

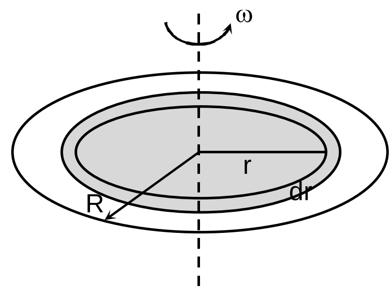
29. 3

Let coefficient of friction = η

Consider element ring of radius r and thickness dr

Let thickness of disc is t.

Viscous force on small elemental ring is



$$dF = \eta(2\pi r dr)\left(\frac{r\omega - 0}{t}\right]$$

$$dF = \frac{\eta.2\pi\omega}{t}.r^2 dr$$

$$d\tau = \overline{r} \times \overline{dF} = \frac{\eta 2\pi\omega}{t} r^3 dr$$

$$\tau = \frac{\eta 2\pi\omega}{t}\int_0^R r^3 dr = \eta\frac{2\pi\omega}{t}.\frac{R^4}{4}$$

$$\tau \propto R^4$$

30. 4

Using straight line equation

$$y = mx + C$$

$$\ln A = \frac{-1}{10}t + 2.6$$

$$A = e^{-\frac{1}{10}t+2.6}$$

$$= e^{-0.1t}.e^{2.6}$$

$$= 12e^{-0.1t}$$

CHEMISTRY

31. 4

$MX \longrightarrow \underset{S}{M^+} + \underset{S}{X^-} \quad \underset{MX}{Ksp} = S^2$

$MX_2 \longrightarrow \underset{S}{\overset{+2}{M}} + 2\underset{2S}{X^-} \quad Ksp_{MX_2} = S \times (2s)^2 = 4S^3$

$MX_3 \longrightarrow \underset{S}{M^{+3}} + 3\underset{3S}{X^-} \quad Ksp_{MX_3} = S \times (3S)^3 = 27S^4$

Solubility (S) of all salts are same

Thus $Ksp_{(MX_3)} < Ksp_{(MX_2)} < Ksp_{(MX)}$

32. 1

Energy released by 'H' atom

$$\Delta E_4 = \frac{3}{4} \times 13.6 \text{ eV}$$

Let He^+ to nth state at the end, so energy required by He^+ ions is

$$\Delta E_{He} = 13.6 \times 4\left(\frac{1}{4} - \frac{1}{n^2}\right)\text{eV}$$

$$\Delta E_H = \Delta E_{He}$$

$$\frac{3}{4} \times 13.6 = 13.6 \times 4\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{3}{16}$$

$$\frac{1}{n^2} = \frac{1}{16} \Rightarrow n = 4$$

33. 4

$2MnO_4^- + 6H^+ + 5H_2O_2 \longrightarrow 2Mn^{+2} + 8H_2O + SO_2 \uparrow$

Now $5H_2O_2 \equiv 2MnO_4$

$\frac{5H_2O_2}{2MnO_4} = \frac{x \text{ moles}}{y \text{ moles}}$; Molarity $KMnO_4 = \frac{y \text{moles}}{\text{vol}} \Rightarrow \frac{1}{25} = \frac{4}{0.005\text{Lt}}$

$\Rightarrow$ y moles $KMnO_4 = 0.0002$

$$x = \frac{5 \times .0002}{2} = 5 \times 10^{-4} \text{ moles}$$

$$\text{Molarity} = \frac{5 \times 10^{-4}}{0.1} = 5 \times 10^{-3}$$

34. 2

35. 4

(a) $2N_2O_5 \longrightarrow 4NO_{2(g)} + O_{2(g)}$

$$\text{Rate} = = -\frac{1}{2}\frac{d(N_2O_5)}{dt} = K_1[N_2O_5]$$

$$= \frac{-d[N_2O_5]}{dt} = 2K_1[N_2O_5], \text{ thus } K = 2K_1$$

(b) $N_2O_5 \longrightarrow 2NO_2 + 1/2\, O_2$

$$\text{Rate} = \frac{-d(N_2O_5)}{dt} = K_2[N_2O_5] \text{ thus } K_2 = K'$$

Thus $2K = K'$

36. 3

37. 1

$Net^{rxn}: Hg_2Cl_2(s) + 2e^- \rightarrow 2Hg^+_{(g)} + 2Cl^-$

Potential of calomel electrode depends on conc. of Cl^- used

$$E = E^o - \frac{.059}{2}\log\left[Cl^-\right]^2$$
$$= E^o - .059\log\left[Cl^-\right]$$

The electrode potential increases with decrease in conc. of Cl^- ions
on dilution by 10 times

$E = E^o + .059\log\left(Cl^-\right)$

38. 3

Our aim is to find

$$H_2 + 2O_2 + S \longrightarrow H_2SO_4 \quad \Delta H_{H_2SO_4} = ?$$

$SO_3 + H_2O \longrightarrow H_2SO_4 \quad \Delta H = -130\ KCal/mol(1)$

$S + O_2 \longrightarrow SO_2 \quad \Delta H = -300\ kcal/mol\ (2)$

$H_2OH + \frac{1}{2}O_2 \longrightarrow H_2O \quad \Delta H = -290\ Kcal/mol\ (3)$

$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3 \quad \Delta H = -100\ Kcal/mol\ (4)$

Adding eq. 1, 2, 3 & 4
We will get

$H_2 + 2O_2 + S \longrightarrow H_2SO_4 \quad \Delta H = -820 Kcal/mol$

39. 1

$\Delta_f GX_{(s)} = -65 Kcal/mol$

$X_{(\ell)} \longrightarrow X(g)$

$K_p = PX_{(g)}$

$\Delta G^o = \Delta_f G^o_{x_g} - \Delta_f GX_{(\ell)}$

$= -60.4 + 65$

$= 4.6\ Kal\ mol^{-1}$

$\Delta G^o = -2.303RT\ \log\ K_f$

$19.228 \times 10^3 = -2.303 \times 8.314 \times 500 \log K_f$

$\log K_p = -2$

$K_p = 10^{-2}$

$K_p = \text{pressure } X_g = 0.01\ atm$

40. 1

41. 2

$46Pd = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^6, 55^0, 4d^{10}$

$\ell = 2$ means 'd' orbital

Total $e^- = 10 + 10 = 20$

42. 3

$D_2O > H_2O \quad \Delta H_f$

$D_2O > H_2O \quad \Delta H_{rap}$

$D_2O > H_2O$ density

$D_2O > H_2O$ mol. mass

43. 1

44. 1

$Li > Na$ reducing rating

$CCl_4 > SiCl_4$ (Boiling pt)

45. 3

CO: $[Ar]4s^2 3d^7$ Magnetic moment will depend

$CO^{+3} : [Ar]3d^6$ upon no. of unpaired e^-

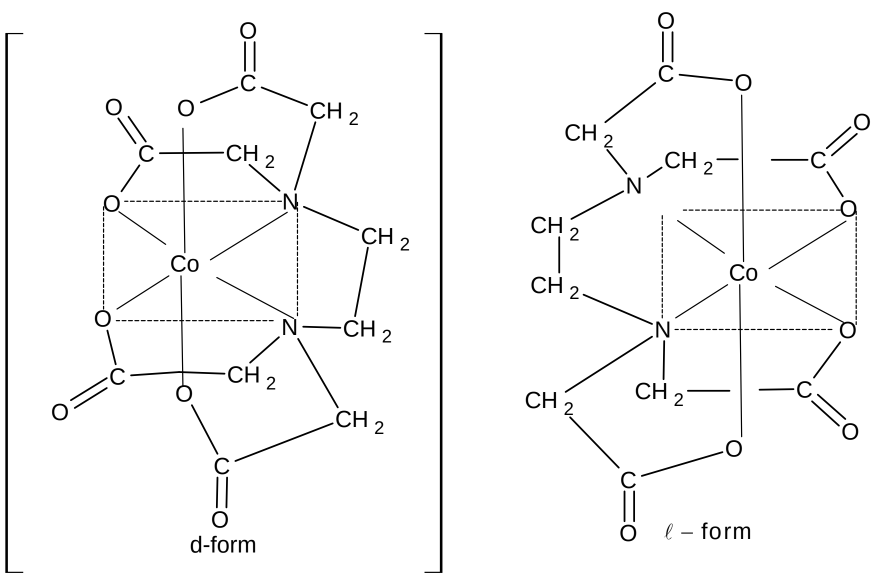
$CO^{+2} [Ar]3d^7$

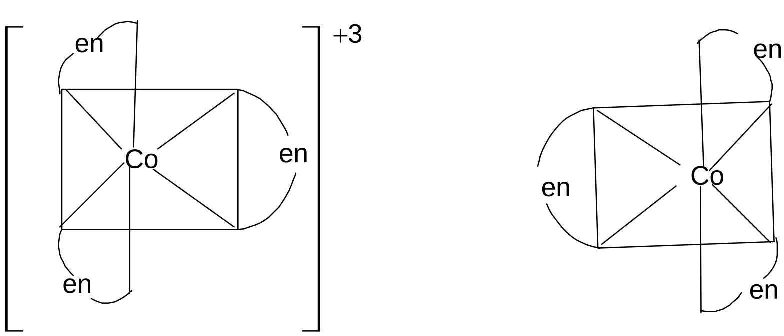
46. 3

SiC has a network solid structure thus hydrolysis will not take place.

47. 3

Optical isomers of $[Co(EDTA)]^-$

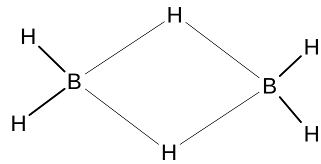


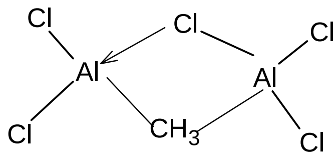


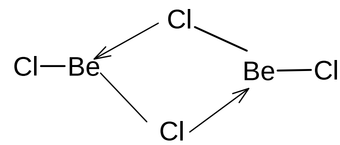
Non supenonposable mirror ionable

48. 3

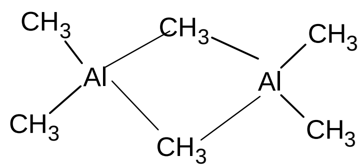
49. 3



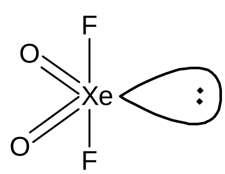




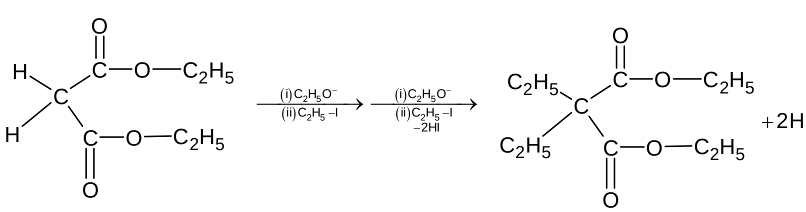
$(BeCl_2)_2 : sp^2 - sp^3 - sp^2$



50. 4



51. 4



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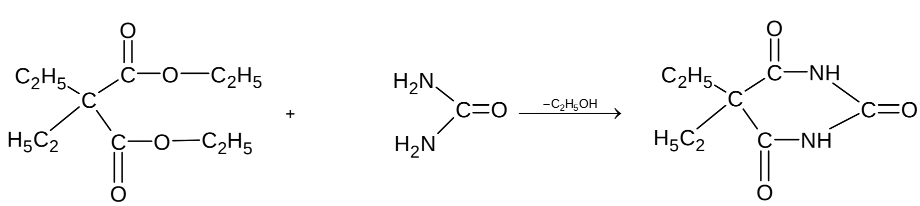
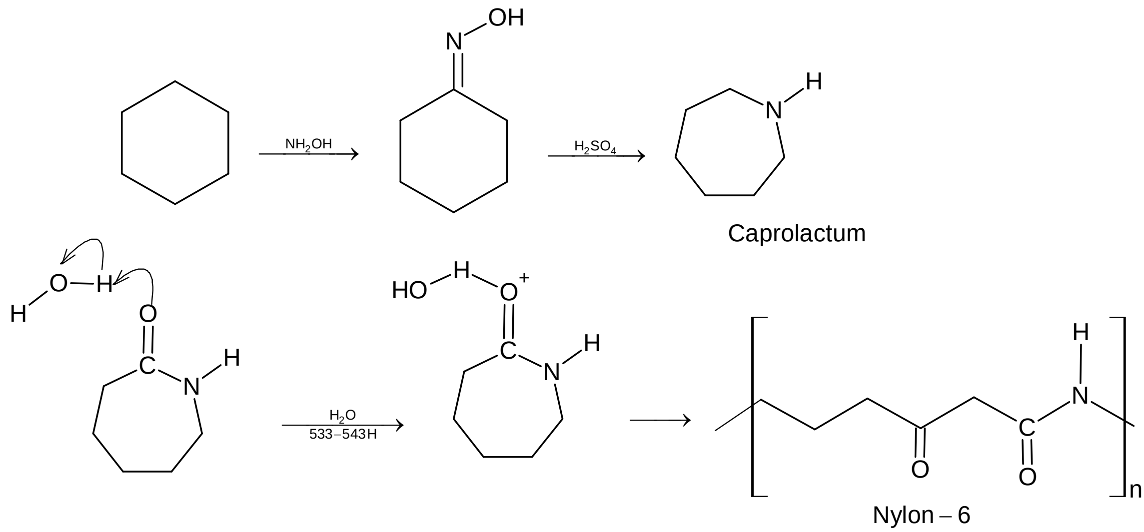


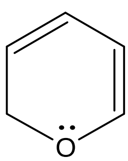
52. 2

53 3

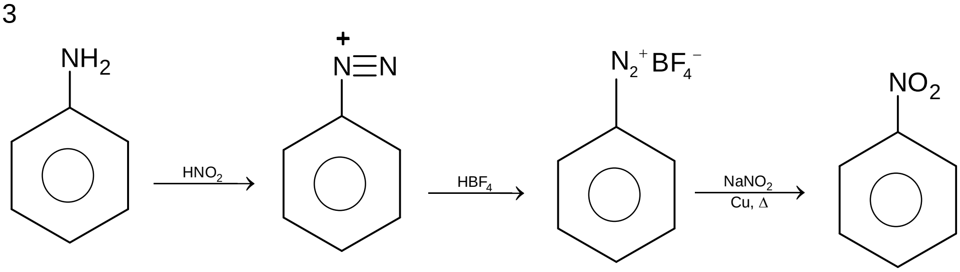


54. 4

Pyran

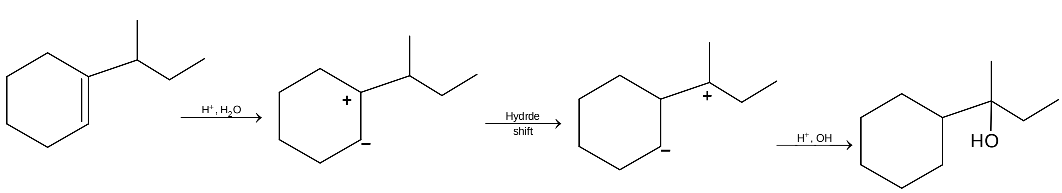


55. 3

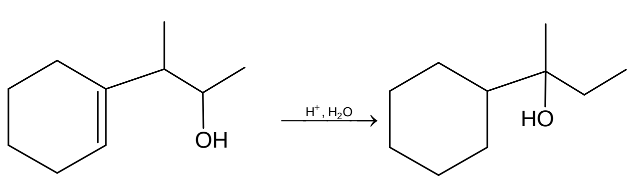


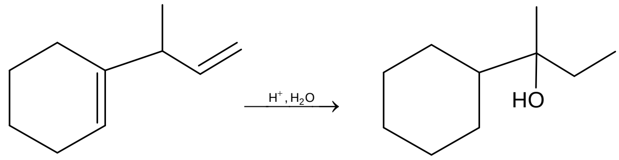
56. 4

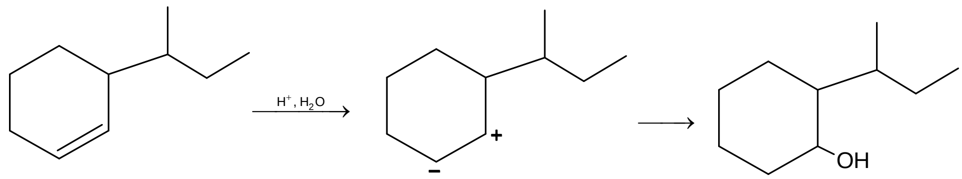
(A)



(B)

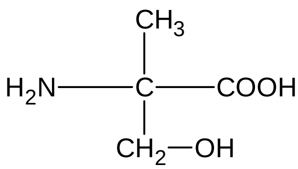


(C) 

(D) 

57. 1

58. 2

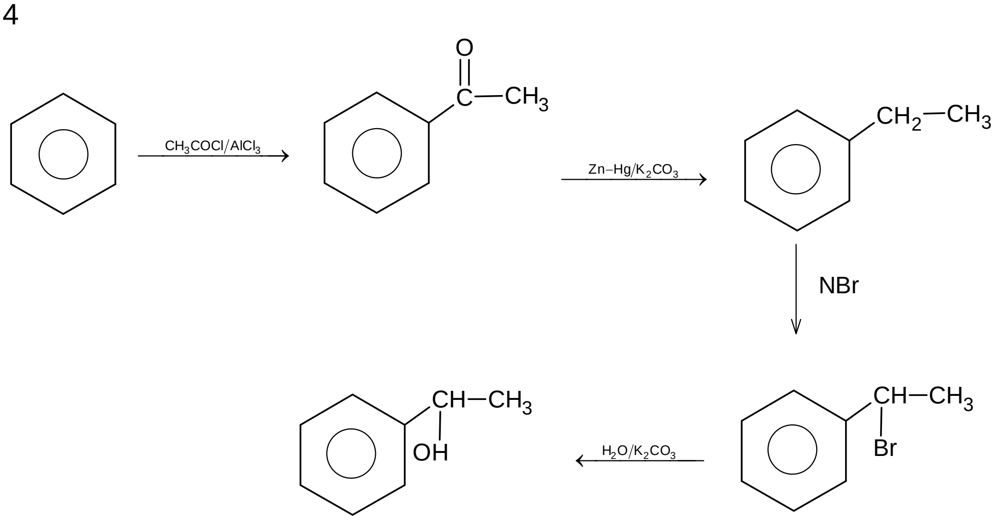


Serine

59. 4

$Ph-NH_2$ cannot be obtained by Gabnel pathalmdi

60. 4



MATHEMATICS

61. 1

AM $\geq$ GM

$$\frac{a+2b+\frac{3c}{2}+\frac{3c}{2}}{4} \geq \left(\frac{9abc^2}{2}\right)^{1/4} \Rightarrow abc^2 \leq \frac{9}{8}$$

62. 3

Equation of tangent at point $(5\cos\theta, 4\sin\theta)$ is $\frac{x\cos\theta}{5}+\frac{y\sin\theta}{4}=1$

$A \equiv (5\sec\theta, 0)$ $B \equiv (0, 4\operatorname{cosec}\theta)$

$0 \equiv (0,0)$

Let (h, k) i.e. the circumcentre

$$\therefore \left(\frac{5}{2h}\right)^2+\left(\frac{4}{2k}\right)^2=1 \Rightarrow \frac{25}{h^2}+\frac{16}{k^2}=4$$

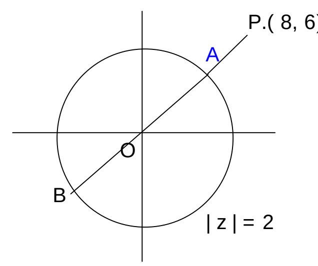
63. 3

$|iz+6-8i|$

$=|i(z-8-6i)|$

$=|z-8-6i|$

$PB = OB + OP = 2 + 10 = 12$



64. 4

$$I=\int_{\frac{1}{2}}^{2}\frac{x^2 \ln x}{\left(1+x^2\right)^3}$$

Put $x=\frac{1}{t} \Rightarrow dx=-\frac{1}{t^2}dt$

$$I=\int_{2}^{\frac{1}{2}}\frac{-\frac{1}{t^2}\ln t\left(-\frac{1}{t^2}\right)dt}{\left(\frac{t^2+1}{t^2}\right)^2}$$

$$=-\int_{\frac{1}{2}}^{2}\frac{t^2 \ln t}{\left(1+t^2\right)^3}=-I$$

$\Rightarrow I = 0$

65. 2

$$\left(4x^2-4x+1\right)+2a(2x-1)=0 \Rightarrow x=\frac{1}{2} \text{ or } \frac{1-2a}{2}$$

$$\cos\theta=\frac{1}{2} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2}=\frac{1-2a}{2}$$

$$\Rightarrow a=\frac{1-\sqrt{3}}{2}$$

$$\Rightarrow a+\sin\theta=\frac{1}{2}$$

66. 3

$(A+B)(A-B)=A^2-B^2 \Rightarrow AB=BA$

$$\Rightarrow \left(A^2BA^{-1}B^{-1}\right)^3=\left(A.ABA^{-1}B^{-1}\right)^3=\left(ABAA^{-1}B^{-1}\right)^3=A^3$$

67. 1

$$h = \frac{|-9+8-1|}{5} = \frac{2}{5} \Rightarrow \frac{\sqrt{3}}{2}a = \frac{2}{5}$$

$$\Rightarrow a = \frac{4}{5\sqrt{3}} \Rightarrow \text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{4\sqrt{3}}{75}$$

68. 1

When $\lambda \in R^+$

$-2\lambda + 5 \le 2\lambda \cos x + 5 \le 2\lambda + 5$

For function to be defined $-2\lambda + 5 \ge 1 \Rightarrow \lambda \le 2$

When $\lambda \in R^-$

$2\lambda + 5 \le 2\lambda \cos x + 5 \le -2\lambda + 5$

$2\lambda + 5 \ge 1 \Rightarrow \lambda \ge -2$

$\lambda = -2, -1, 0, 1, 2$

69. 3

$$\left(\frac{d^3y}{dx^3} + y\frac{dy}{dx}\right)^7 = \left(x^3 . \frac{d^2y}{dx^2}\right)^5$$

$m = 3, n = 7 \Rightarrow m + n = 10$

70. 3

$a^2 + 4ah = 48$

$$v = a^2h = a\left(\frac{48 - a^2}{4}\right) = \frac{1}{4}\left(48a - a^3\right)$$

$$\frac{dv}{da} = \frac{1}{4}\left(48 - 3a^2\right) = 0 \Rightarrow a = 4 \text{ ,} h = 2$$

$v_{max} = 32 \text{ m}^3$

71. 1

$$f'(x) = \frac{2}{x} - \frac{x.|x|}{x} - |x| = \frac{2}{x} - 2|x|$$

$$f'(x) > 0 \Rightarrow \frac{2}{x} - 2|x| > 0 \Rightarrow x \in (0,1)$$

72. 3

$$\sum_{i=1}^{5} xi = 55 \Rightarrow \bar{x} = 11$$

$$\sigma^2 = \frac{\sum_{i=1}^{5}(2x_i + 7)^2}{5} - \left(\frac{\sum_{i=1}^{5}(2x_i + 7)}{5}\right)^2$$

$= 857 - 841 = 16 \Rightarrow \sigma = 4$

73. 1

$$\cos\theta = \frac{2 - 1 + 3}{\sqrt{6}\sqrt{11}} = \frac{4}{\sqrt{66}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{\sqrt{66}}\right)$$

74. 3

$$a + b + c = \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z} = 1$$

75. 4

$$y = \frac{\cos x}{\sin^2 x(\cos x - \sin x)} = \frac{1 + \cos 2x + \sin 2x}{(1 - \cos 2x)\cos 2x}$$

$$\frac{dy}{dx} = 0 \text{ at } x = \frac{\pi}{8}$$

$$y_{max} = \frac{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\left(1 - \frac{1}{\sqrt{2}}\right) \times \frac{1}{\sqrt{2}}} = 6 + 4\sqrt{2} < 12$$

76. 4

$$(x+1)^{2016} = \sum_{r=0}^{2016} {}^{2016}C_r x^{2016-r} = \sum_{r=0}^{2016} T_r$$

Put $x = ix$

$$(ix+1)^{2016} = \sum_{r=0}^{2016} {}^{2016}C_r (ix)^{2016-r}$$

$$\Rightarrow (T_0 - T_2 + T_4 - .... + T_{2016})^2 + (T_1 - T_3 + T_5.....T_{2015})$$

$$= (x^2 + 1)^{2016}$$

77. 4

$$\int_1^{6\pi} \left(\left[\sec^{-1} x\right] + \left[\cot^{-1} x\right]\right) dx$$

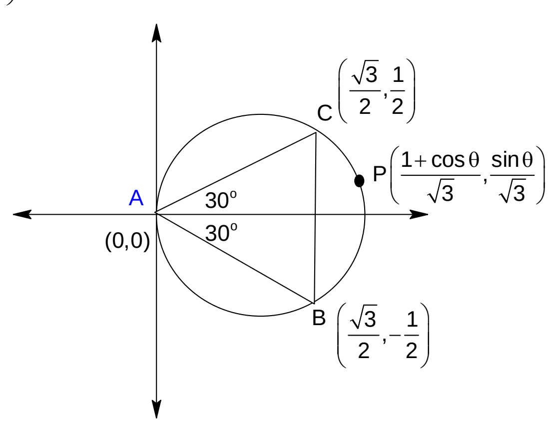
$$= \int_1^{6\pi} \left[\sec^{-1} x\right] dx + 0 = \int_1^{\sec 1} \left[\sec^{-1} x\right] dx + \int_{\sec 1}^{6\pi} \left[\sec^{-1x}\right] dx$$

$$0 + \int_1^{6\pi} 1 dx = 6\pi - \sec 1$$

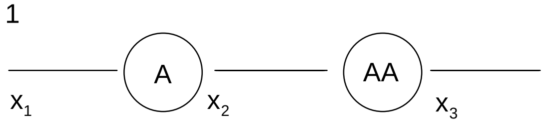
78. 3

$$PA^2 + PB^2 + PC^2$$

$$= \left(\frac{1+\cos\theta}{\sqrt{3}}\right)^2 + \left(\frac{\sin\theta}{\sqrt{3}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{3}} - \frac{1}{2\sqrt{3}}\right)^2 + \left(\frac{\sin\theta}{\sqrt{3}} - \frac{1}{2}\right)^2 + \left(\frac{\cos\theta}{\sqrt{3}} - \frac{1}{2\sqrt{3}}\right)^2 + \left(\frac{\sin\theta}{\sqrt{3}} + \frac{1}{2}\right)^2 = 2$$

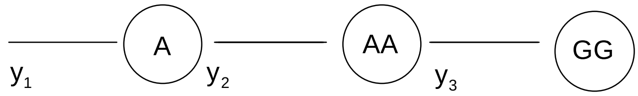


79. 1



$x_1 + x_2 + x_3 = 5$, $15 \times 2 \times \frac{5!}{2!} = 1800$

$0 \le x_1, x_3 \le 4, 1 \le x_2 \le 5$



$y_1 + y_2 + y_3 = 4, 0 \le y_1, y_2 \le 3$, $1 \le y_2 \le 4$

${}^5C_2 \times 2 \times 4!$,

Required $= 1800 - 480 = 1320$

80. 3

p	q	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	~q	P^~q	pv~q	~(pv~q)	(p^~q)v ~(pv~q)	q	q	p→q	p→(p→q)
T	T	T	F	F	F	T	F	F	T	T	T	T
T	F	F	T	T	T	T	F	T	T	F	F	F
F	T	F	T	F	F	F	T	T	F	T	T	T
F	F	T	F	T	F	T	F	F	F	F	T	T

81. 2

$P(x_1, y_1) \Rightarrow x_1 y_1 = 100$

Equation of tangent $xy_1 + x_1 y = 200$

Area = $\frac{1}{2} \times \frac{200 \times 200}{x_1 y_1} = 200$

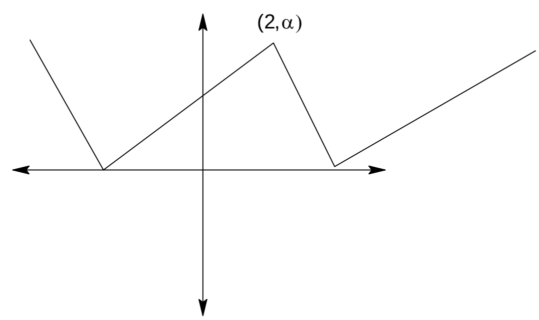
82. 1

$5R, 4G, 3B$

$\frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} \times \frac{1}{9} = \frac{1}{12}$

83. 1

$\alpha - 5 > 0 \Rightarrow \alpha > 5$



84. 2

Let $\left[\frac{1}{|x|}\right] = n$

$\Rightarrow n \le \frac{1}{|x|} < n + 1 \Rightarrow \frac{1}{n+1} < |x| \le \frac{1}{x}$

$\Rightarrow \frac{1}{(n+1)^2} < x^2 \le \frac{1}{n^2}$

$\frac{n(n+1)}{2(n+1)^2} < \frac{x^2 n(n+1)}{2} \le \frac{n(n+1)}{2n^2}$

$\lim_{x \to 0} x^2 \left(1 + 2 + 3 + ... + \left[\frac{1}{|x|}\right]\right)$

$\lim_{n \to \infty} x^2 . \frac{n(n+1)}{2}$

Using sandwich theorem both LHL and RHL is $\frac{1}{2}$

85. 3

$$LHL = \lim_{h\to 0}\frac{\ln\left(1+sgn(-h)+\{-h\}^2\right)}{1-\cos\{-h\}}$$

$$= \lim_{h\to 0}\frac{sgn(1-1+(1-h))}{1-\cos(1-h)} = 0$$

$$RHL = \lim_{h\to\infty}\frac{\ln\left(1+sgnh+\{h\}^2\right)}{1-\cos\{h\}} = \lim_{h\to 0}\frac{\ln\left(1+1+h^2\right)}{1-\cosh} = \infty$$

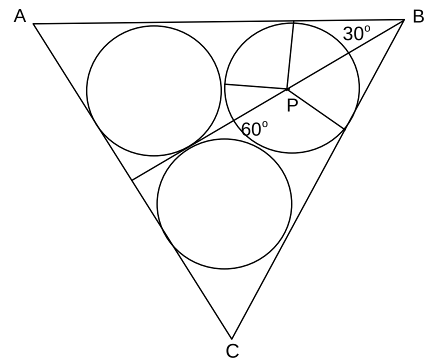
Non-removable discontinuity.

86. 2

$R = OA = OP + PA$

$= \sec 30^o + \sec 60^o$

$= \frac{2}{\sqrt{3}} + 2 = 2\left(1+\frac{1}{\sqrt{3}}\right)$



87. 1

$x^{2016} - x^{2015} + x^{1008} + x^{1003} + 1$ is always positive $\forall x \in R$

Hence no solution.

88. 3

$a_1 a_2 a_3 ....a_{12} = 8^{2014}$

$\Rightarrow a^{12} r^{66} = 2^{3\times 2014} \Rightarrow (a_1^2)(r^{11}) = (2^2)^{\alpha}(2^{11})^{\beta} = 2^{1007}$

$\beta = 1, 3, 5, 7........91$

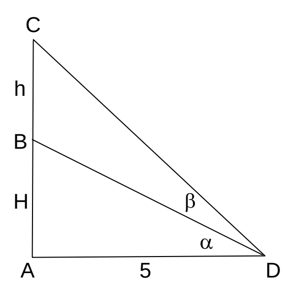
$\therefore$ no of ordered pairs of $(a_1, r) = 46$

89. 1

$\tan(\alpha+\beta) = \frac{h+h}{5}, \tan\alpha = \frac{H}{5}$

$2\tan\beta\cot\alpha = 1 \Rightarrow \tan\beta = \frac{H}{10}$

$$\Rightarrow \frac{\frac{H}{5}+\frac{H}{10}}{1-\frac{H^2}{50}} = \frac{h+H}{5} \Rightarrow H^3 + hH^2 + 25H - 50h = 0$$



90. 2

All rays coming from the positive direction of x-axis and parallel to the axis of parallels after reflection passes through the focus of the parabola.